Parameter Estimation, Slow Roll and the Inflationary Potential

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Theoretical Uncertainties in Inflationary Predictions

\[ V \propto \exp(\phi/\mu) \]
\[ V \propto \phi^p \]
\[ V \propto 1 - (\phi/\mu)^2 \]
\[ V \propto 1 - (\phi/\mu)^p \quad (p > 2) \]

\[ N \simeq 60 + \frac{1}{6} \ln(-n_T) + \frac{1}{3} \ln(T_{RH}/10^{16}\text{GeV}) - \frac{1}{3} \ln \gamma \]

- Reheat temperature can vary from GUT scale \((10^{15} \text{ GeV})\) to nucleosynthesis scale \((1 \text{ MeV})\)

- Resulting uncertainty in \(N\) about 14 e-folds gives uncertainties in observables:
  \[ \frac{\Delta r}{r} \sim 1 \]
  \[ \Delta n \sim 0.02 \]

Measuring these observables from the data may not be the best way to learn about inflation!
Hubble Slow Roll Formalism for Single Field Inflation: Review

Assume Hubble parameter during inflation is a function of field rather than time (i.e. field is monotonic in time). Leads to Hamilton-Jacobi Equation:

$$[H'(\phi)]^2 - \frac{12\pi}{m_{P1}^2} H^2(\phi) = -\frac{32\pi^2}{m_{P1}^4} V(\phi)$$

Define HSR parameters:

$$\epsilon(\phi) \equiv \frac{m_{P1}^2}{4\pi} \left[ \frac{H'(\phi)}{H(\phi)} \right]^2$$

$$\ell \lambda_H \equiv \left( \frac{m_{P1}^2}{4\pi} \right)^{\ell} \frac{(H')^{\ell-1}}{H^{\ell}} \frac{d^{(\ell+1)} H}{d\phi^{(\ell+1)}}; \; \ell \geq 1$$

Then the potential is given by:

$$H^2(\phi) \left[ 1 - \frac{1}{3} \epsilon(\phi) \right] = \left( \frac{8\pi}{3m_{P1}^2} \right) V(\phi)$$

Evolution of HSR parameters described by an infinite set of coupled first order DEs. In practice, must truncate this series at order $M$:

\[
\begin{align*}
\frac{d^{M+1} \lambda_H}{d\phi^{M+1}} &= 0 \\
\Rightarrow \frac{d^{(M+2)} H}{d\phi^{(M+2)}} &= 0
\end{align*}
\]

This leads to an analytic solution for the motion:

\[
H(\phi) = H_0 \left[ 1 + A_1 \left( \frac{\phi}{m_{Pl}} \right) + \ldots + A_{M+1} \left( \frac{\phi}{m_{Pl}} \right)^{M+1} \right]
\]

\[
\epsilon(\phi) = \frac{m_{Pl}^2}{4\pi} \left[ \frac{(A_1/m_{Pl}) + \ldots + (M + 1) (A_{M+1}/m_{Pl}) (\phi/m_{Pl})^M}{1 + A_1 (\phi/m_{Pl}) + \ldots + A_{M+1} (\phi/m_{Pl})^{M+1}} \right]^2
\]

\[
A_1 = \sqrt{4\pi \epsilon_0}
\]

\[
A_{\ell+1} = \frac{(4\pi)^\ell \ell \lambda_{H,0}}{(\ell + 1)! A_1^{\ell-1}} \quad \ell \geq 1
\]

Liddle (2003), Ramirez & Liddle (2005)
Hubble Slow Roll Formalism as Potential Generator for Markov Chain-based Parameter Estimation

Algorithm:

- Pick HSR parameters at fiducial wavenumber corresponding to phi=0
- Calculate k as a function of phi
- Use analytic solutions to H, epsilon and eta to calculate the primodial power spectra
- Feed into e.g. CAMB to calculate CMB power spectra
- Models where inflation ends within the k-range accessible by CMB and LSS are rejected
- Have option of applying an a posteriori “sufficient e-folds” prior

\[
\frac{d\phi}{d \ln k} = - \frac{m_{Pl}}{2\sqrt{\pi}} \frac{\sqrt{\epsilon}}{1 - \epsilon}
\]

\[
P_R = \frac{[1 - (2C + 1)\epsilon + C\eta]^2}{\pi\epsilon} \left( \frac{H}{m_{Pl}} \right)^2 \bigg|_{k=aH}
\]

\[
P_h = [1 - (C + 1)\epsilon]^2 \frac{16}{\pi} \left( \frac{H}{m_{Pl}} \right)^2 \bigg|_{k=aH}
\]

\[
A_s = \frac{[1 - (2C + 1)\epsilon_0 + C\eta_0]^2}{\pi\epsilon_0} \left( \frac{H_0}{m_{Pl}} \right)^2
\]

Difference from HSR Monte Carlo Simulation

- Observables to second order in slow roll from $M=8$ HSR simulation

- The initial ranges for the HSR parameters are chosen “by hand” in this simulation

- However, in the reconstruction technique, we let the data determine both their ranges, and the number $M$ which a given data set can effectively constrain

Peiris et al. (2003)
Case I

HSR parameters used to construct power spectra indirectly by calculating standard observables to second order in slow roll
Case II: Preliminary Results

HSR parameters used to construct power spectra directly
Case II
HSR

parameter estimation
with standard
observables

HSR
Case I

HSR
Case II
Reconstructed Potential

HSR parameters used to construct power spectra indirectly by calculating standard observables to second order in slow roll

PRELIMINARY RESULTS