Kavli Institute
for Cosmological Physics
at The University of Chicago

Parameter Estimation, Slow Roll and the Inflationary Potential

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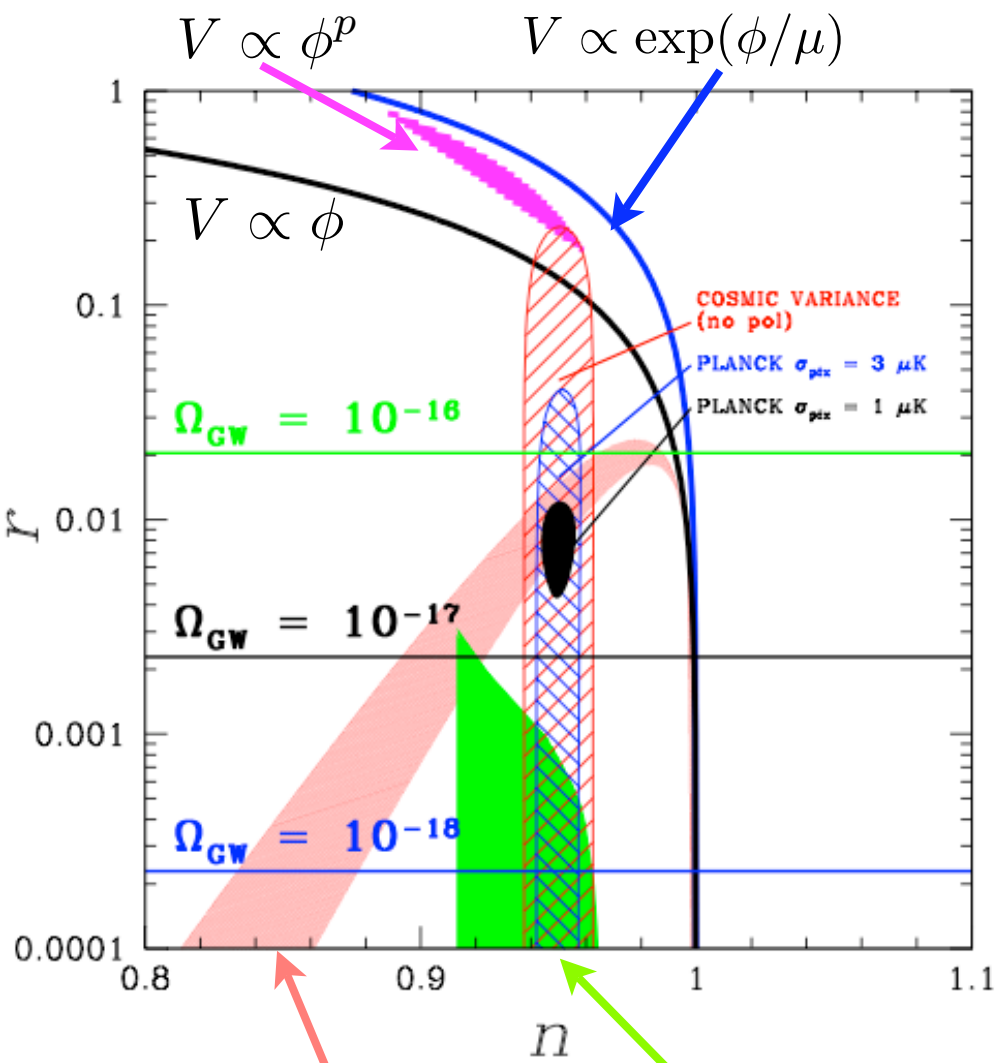
Schramm Symposium: "New Views of the Universe"
8-13 Dec 2005

Theoretical Uncertainties in Inflationary Predictions

$$N \simeq 60 + \frac{1}{6}\ln(-n_T) + \frac{1}{3}\ln(T_{RH}/10^{16}\mathrm{GeV}) - \frac{1}{3}\ln\gamma$$



Kinney & Riotto (astro-ph/0511127)

- Reheat temperature can vary from GUT scale (10^{15} GeV) to nucleosynthesis scale (1 MeV)

- Resulting uncertainty in N about 14 efolds gives uncertainties in observables:

$$\frac{\Delta r}{r} \sim 1$$

$$\Delta n \sim 0.02$$

Measuring these observables from the data may not be the best way to learn about inflation!

Hubble Slow Roll Formalism for Single Field Inflation: Review

Assume Hubble parameter during inflation is a function of field rather than time (i.e. field is monotonic in time). Leads to Hamilton-Jacobi Equation:

$$[H'(\phi)]^2 - \frac{12\pi}{m_{\rm Pl}^2} H^2(\phi) = -\frac{32\pi^2}{m_{\rm Pl}^4} V(\phi)$$

Define HSR parameters:

$$\epsilon(\phi) \equiv \frac{m_{\rm Pl}^2}{4\pi} \left[\frac{H'(\phi)}{H(\phi)}\right]^2$$

$${}^{\ell}\lambda_H \equiv \left(\frac{m_{\rm Pl}^2}{4\pi}\right)^{\ell} \frac{(H')^{\ell-1}}{H^{\ell}} \frac{d^{(\ell+1)}H}{d\phi^{(\ell+1)}}; \ \ell \geq 1$$

Then the potential is given by:

$$H^2(\phi)\left[1 - \frac{1}{3}\epsilon(\phi)\right] = \left(\frac{8\pi}{3m_{\rm Pl}^2}\right) V(\phi)$$

Barrow, Parsons & Liddle (1994), Hoffman & Turner (2001), Kinney (2002), Kinney & Easther (2003)

Evolution of HSR parameters described by an infinite set of coupled first order DEs. In practice, must truncate this series at order M:

$$^{M+1}\lambda_H = 0$$

$$\Rightarrow \frac{d^{(M+2)}H}{d\phi^{(M+2)}} = 0$$

This leads to an analytic solution for the motion:

$$H(\phi) = H_0 \left[1 + A_1 \left(\frac{\phi}{m_{\rm Pl}} \right) + ... + A_{M+1} \left(\frac{\phi}{m_{\rm Pl}} \right)^{M+1} \right]$$

$$\epsilon(\phi) = \frac{m_{\rm Pl}^2}{4\pi} \left[\frac{(A_1/m_{\rm Pl}) + ... + (M+1)\,(A_{M+1}/m_{\rm Pl})\,(\phi/m_{\rm Pl})^M}{1 + A_1\,(\phi/m_{\rm Pl}) + ... + A_{M+1}\,(\phi/m_{\rm Pl})^{M+1}} \right]^2$$

$$A_1 = \sqrt{4\pi\epsilon_0}$$

$$A_{\ell+1} = \frac{(4\pi)^\ell \; {}^\ell\lambda_{H,0}}{(\ell+1)!\; A_1^{\ell-1}}; \;\; \ell \geq 1$$

Liddle (2003), Ramirez & Liddle (2005)

Hubble Slow Roll Formalism as Potential Generator for Markov Chain-based Parameter Estimation

Algorithm:

- Pick HSR parameters at fiducial wavenumber corresponding to phi=0
- Calculate k as a function of phi
- Use analytic solutions to H, epsilon and eta to calculate the primodial power spectra
- Feed into e.g. CAMB to calculate CMB power spectra
- Models where inflation ends within the k-range accessible by CMB and LSS are rejected
- Have option of applying an a posteriori "sufficient e-folds" prior

$$\frac{d\phi}{d\ln k} = -\frac{m_{\rm Pl}}{2\sqrt{\pi}}\frac{\sqrt{\epsilon}}{1-\epsilon}$$

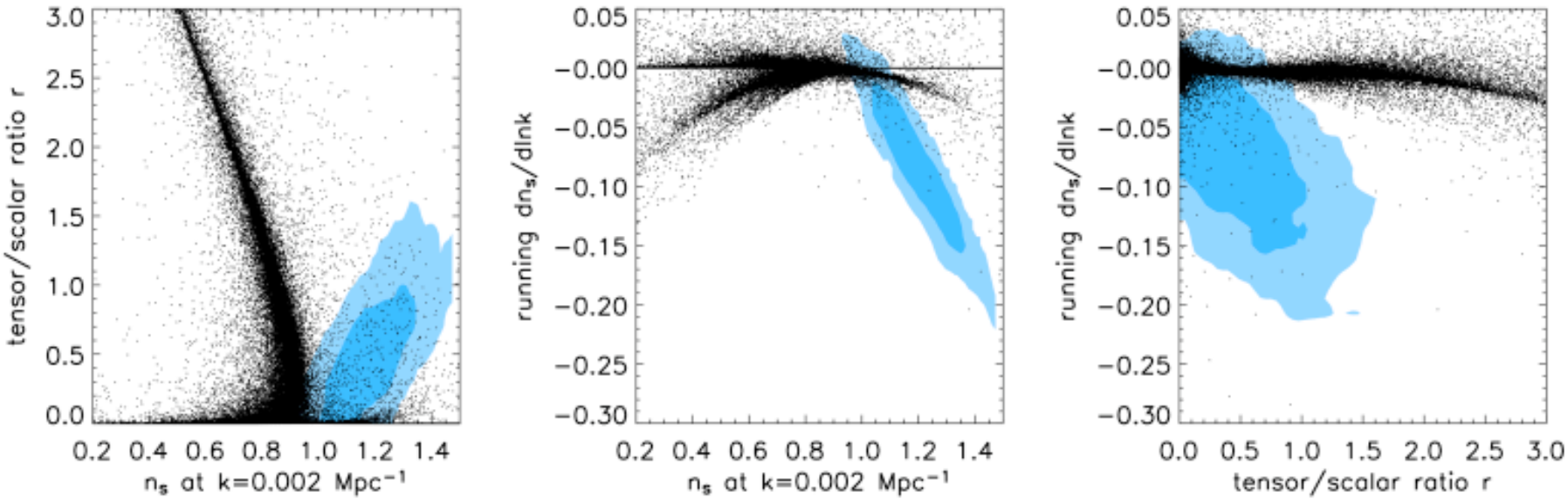
$$P_{\mathcal{R}} = \frac{[1-(2C+1)\epsilon + C\eta]^2}{\pi\epsilon}\left(\frac{H}{m_{\rm Pl}}\right)^2 |_{k=aH}$$

$$P_h = [1-(C+1)\epsilon]^2 \frac{16}{\pi}\left(\frac{H}{m_{\rm Pl}}\right)^2 |_{k=aH}$$

$$A_s = \frac{[1-(2C+1)\epsilon_0 + C\eta_0]^2}{\pi\epsilon_0}\left(\frac{H_0}{m_{\rm Pl}}\right)^2$$

Lidsey et al (1995)

Difference from HSR Monte Carlo Simulation



Peiris et al. (2003)

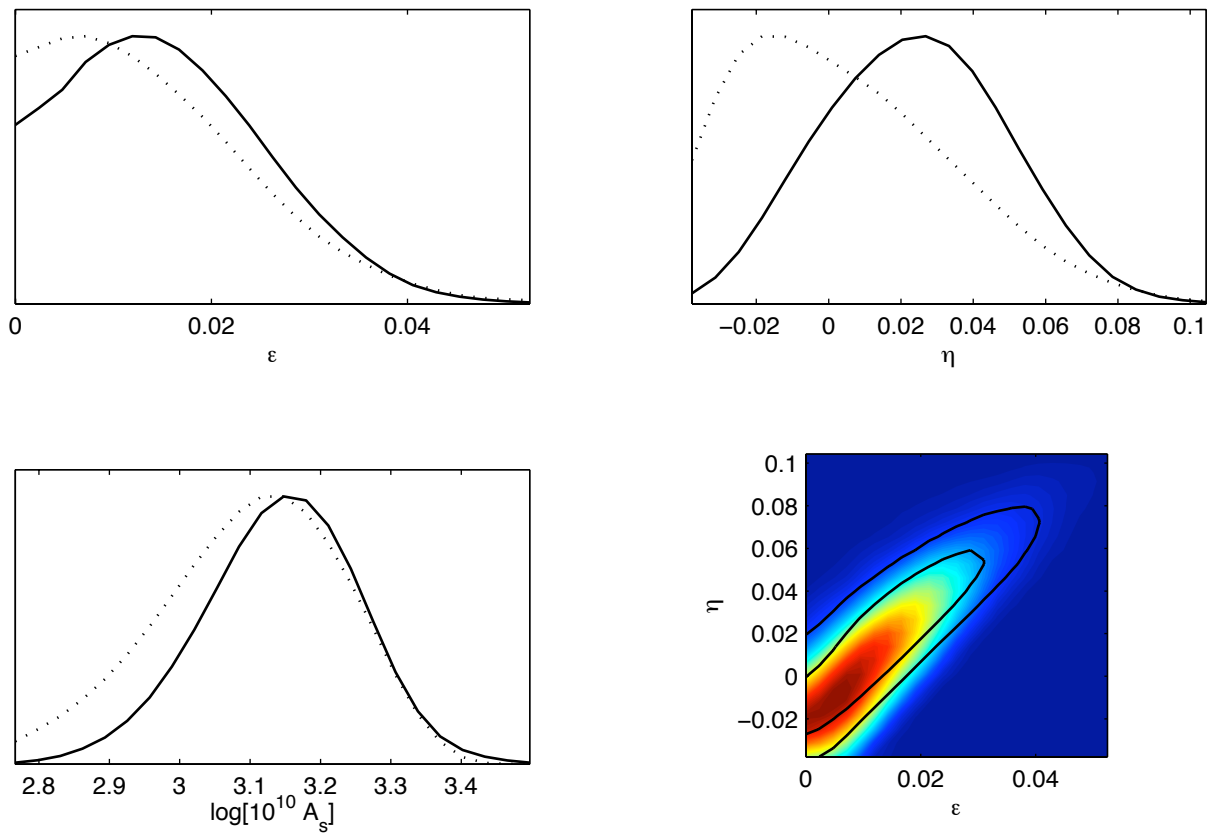
- Observables to second order in slow roll from M=8 HSR simulation

- The initial ranges for the HSR parameters are chosen "by hand" in this simulation

- However, in the reconstruction technique, we let the data determine both their ranges, and the number M which a given data set can effectively constrain

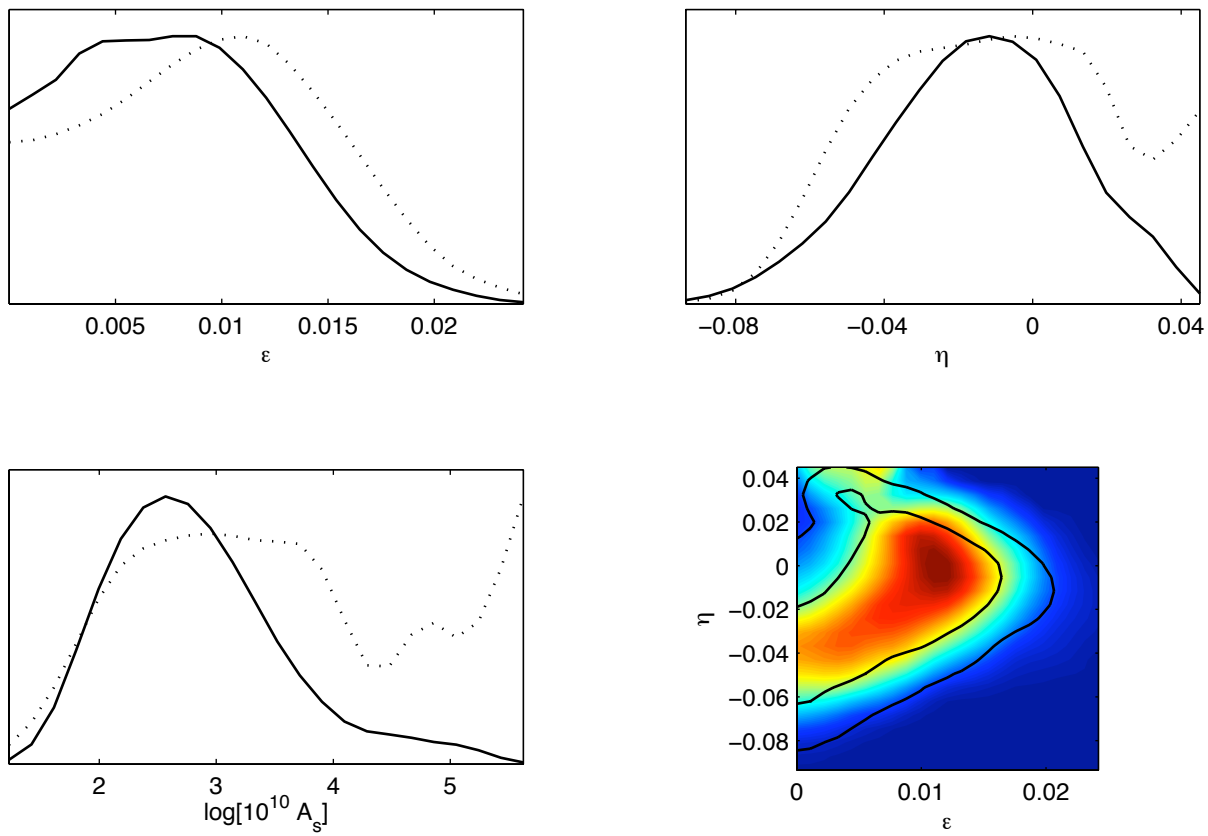
Case I

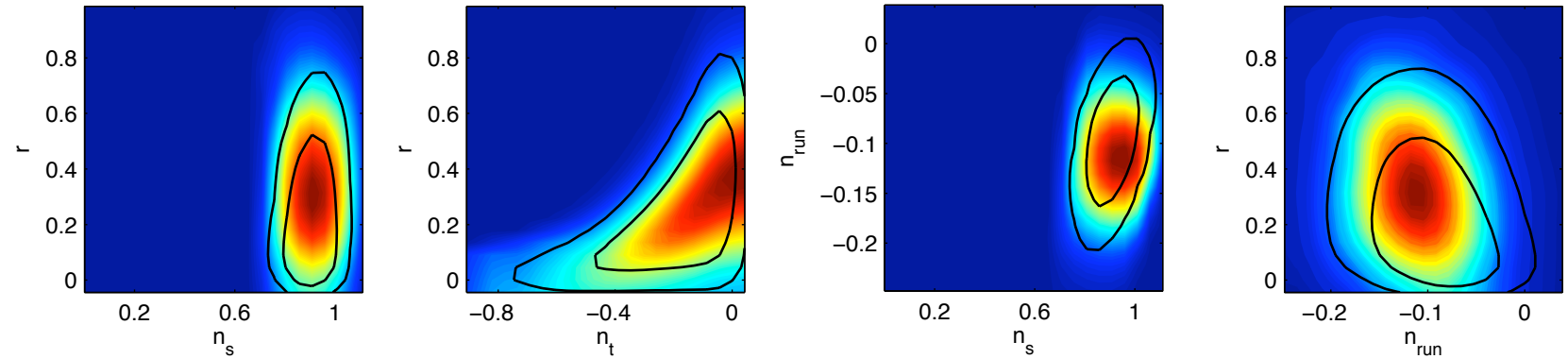
HSR parameters used to construct power spectra indirectly by calculating standard observables to second order in slow roll



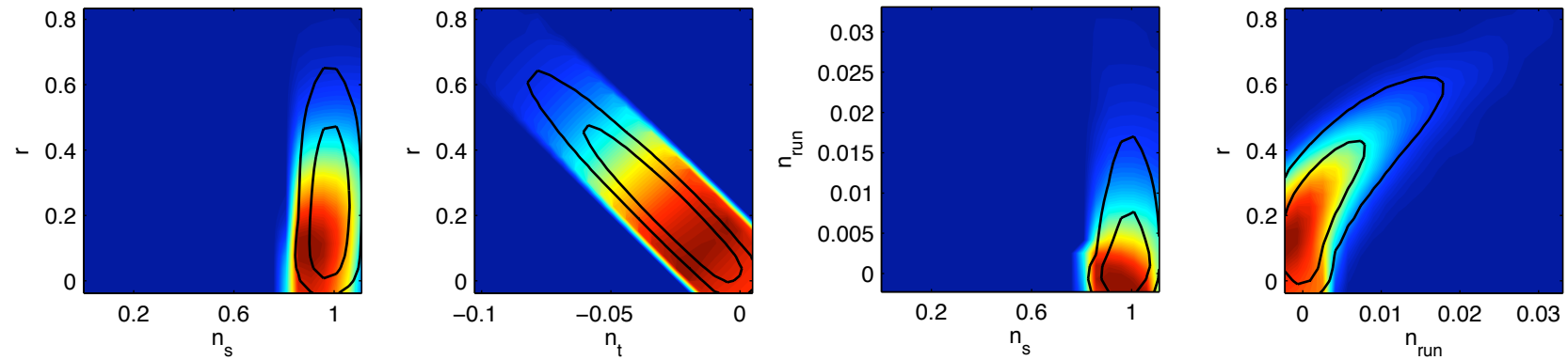
Case II: Preliminary Results

HSR parameters used to construct power spectra directly

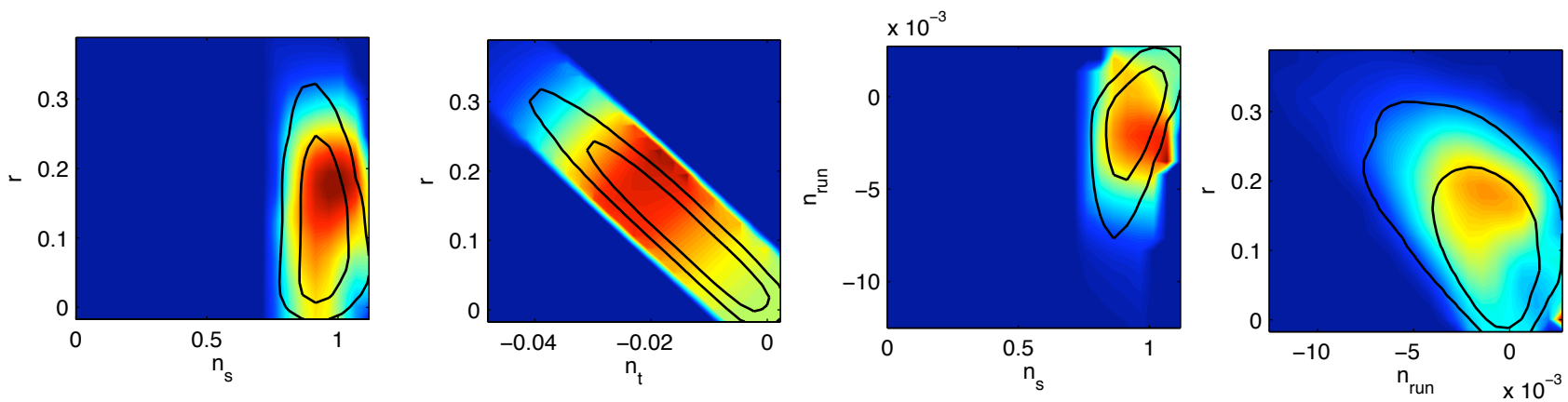




parameter estimation with standard observables

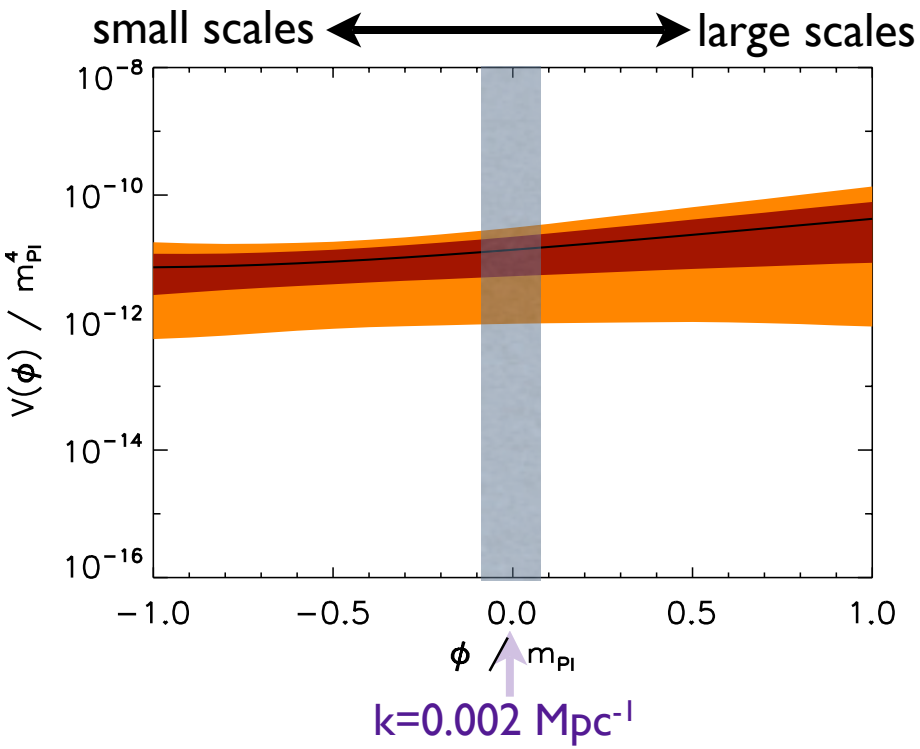


HSR Case I

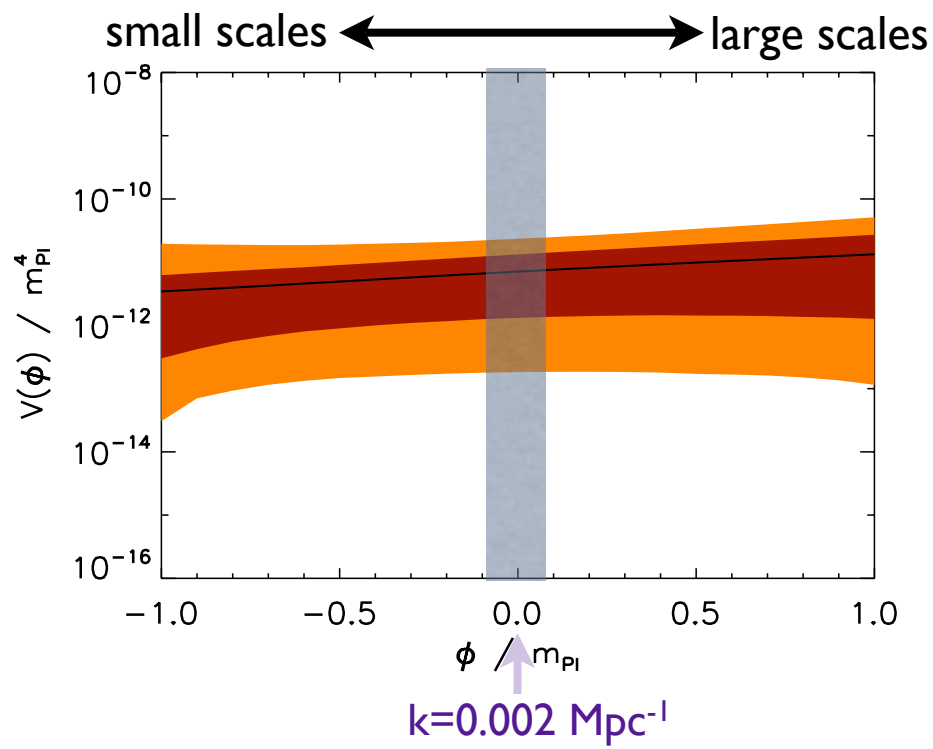


HSR Case II

Reconstructed Potential



HSR parameters used to construct power spectra indirectly by calculating standard observables to second order in slow roll



HSR parameters used to construct power spectra directly

PRELIMINARY RESULTS