An effective theory of initial conditions in inflation

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expanding: hep-th/0507081
back-reaction: hep-th/0512xxx

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We begin with a simple question

Why are we able to explain what happens at long distances without knowing what happens at short distances?

In quantum field theory we have an answer: the details at short distances do not matter . . . at least not much!

$\Rightarrow$ decoupling & effective field theory

Overview

- Primordial perturbations in inflation
- The trans-Planckian problem of inflation
- An effective theory of initial conditions
- Boundary renormalization
- Observational outlook and conclusions

Primordial perturbations from inflation

- **Let us briefly review the origin of primordial perturbations in inflation**

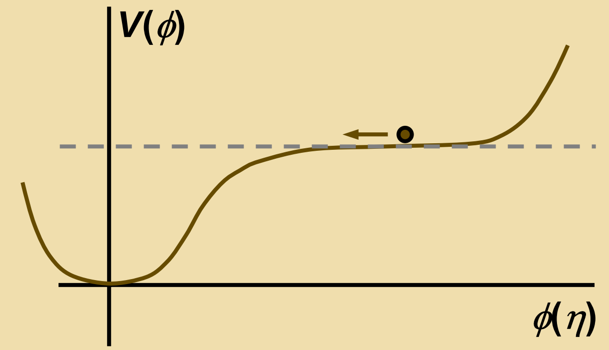
- **In quantum field theory, there is always some inherent variation in a field, $\varphi\eta,$□.**
  - **The pattern of fluctuations is then characterized by the variance of φ**

- **To calculate the variance, expand the field in its operator eigenmodes**

$$\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3}\left[\varphi_k e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \varphi_k^* e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^\dagger\right]$$

- **The Fourier transform of the variance is the power spectrum**

- **The time-dependent eigenmode $\varphi_k(\eta)$ satisfies the Klein-Gordon equation**
  - **one constant of integration is fixed by equal-time commutation relation**
  - **but how do we choose the other, f_k?**



$$\langle 0|\varphi(\eta,\mathbf{x})\varphi(\eta,\mathbf{y})|0\rangle \neq 0$$

$$\langle 0|\varphi(\eta,\mathbf{x})\varphi(\eta,\mathbf{y})|0\rangle = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} \frac{2\pi^2}{k^3} P_k^{\varphi}(\eta)$$

$$\varphi_k = \frac{\varphi_k^0 + f_k \varphi_k^0{}^*}{\sqrt{1 - f_k f_k^*}}$$

Choosing the vacuum state

- **At very short distances, << 1/*H*, the background curvature is not very apparent and space-time looks flat**
- **Therefore a natural choice is the state that matches with the flat space vacuum as $k \to \infty$ with η fixed; this choice fixes $f_k = 0$**
- **At some stage we might worry about some of our underlying assumptions**
  - $H \ll k \ll M_{pl}$
  - **sometimes η is taken to ∞**
  - **complicated dynamics/other fields**
- **We have encountered the question posed at the very beginning:**
  - **how do we know what happens at very short length scales (or any scale < $1/M_{pl}$)?**
- **If we assume that—to some degree—these details decouple, the leading result should be that given by this "vacuum"**

de Sitter example:

$$\varphi_k = \varphi_k^0 = \frac{iH}{k\sqrt{2k}}(1+ik\eta)e^{-ik\eta}$$

$$P_k^\varphi(\eta) = \frac{k^3}{2\pi^2}\left|\varphi_k^0\right|^2$$

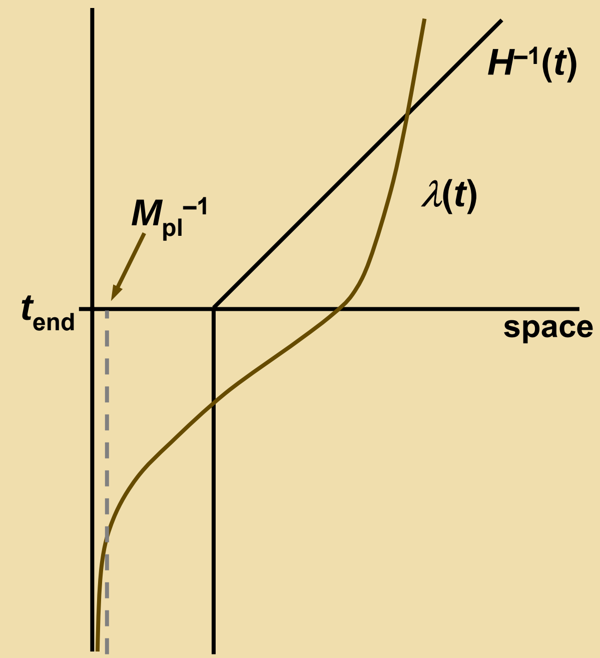
$$P_k^\varphi(k\eta \to 0) = \frac{H^2}{4\pi^2}$$

a flat primordial power spectrum

This behavior is more or less observed in the CMB; so to *leading order*, choosing of the standard vacuum seems to have been justified

The trans-Planckian problem

- We would like to be able to calculate the corrections to this leading result, but there is a subtlety to decoupling during inflation
- The expansion of the background means that what may be a large scale in the primordial background was smaller and smaller the earlier we follow it back during inflation
- So some perturbation that produces, for example, a feature in the CMB was much smaller when it arose during inflation
  - 60–70 *e*-folds to solve the horizon problem
  - a bit more and the wavelength of that mode would have been smaller than the Planck length at some time
- What we need is an effective theory description of the possible differences between our assumed vacuum state and the true vacuum
  - Collins & Holman, 2005
  - Greene, Schalm, Shiu & van der Schaar, 2004–2005



- Brandenberger & J. Martin, 2001–2003
- Easther, Greene, Kinney & Shiu, 2001–2002
- Niemeyer & Kempf, 2001
- Danielsson, 2002
- Goldstein & Lowe, 2003
- Collins & M. Martin, 2004
- Kaloper, Kleban, Lawrence, Shenker & Susskind, 2002
- Burgess, Cline, Lemieux & Holman, 2003

An effective initial state—boundary conditions

- **Let us return to the point where we chose a particular initial state**

- **We shall examine the case of flat space**
  - **the regime in which the new effects will appear should be at much shorter lengths than the Hubble horizon**
  - **FRW case is in hep-th/0507081**

- **Earlier we mentioned that a state is defined up to one *k*-dependent constant of integration**

- **Let us define our state by imposing an initial condition at $t = t_0$ and evolve forward**
  - **Notice that this initial condition includes the standard vacuum state, $_k$ =**

- **In an effective theory, there is always an inherent error between predictions based on our theory and those of a better description of nature**
  - **e.g. Feynman–Gell-Mann (*V* – *A*) theory compared with electroweak theory**

$$S = \int d^4x \left[\tfrac{1}{2} \partial_\mu \varphi \partial_\nu \varphi - \tfrac{1}{2} m^2 \varphi^2 \right]$$

$$\varphi = \int \frac{d^3\mathbf{k}}{(2\pi)^3} \left[\varphi_k e^{i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}} + \varphi_k^* e^{-i\mathbf{k}\cdot\mathbf{x}} a_{\mathbf{k}}^\dagger \right]$$

$$\ddot{\varphi}_k = -\omega_k^2 \varphi_k, \qquad \omega_k = \sqrt{k^2 + m^2}$$

$$\dot{\varphi}_k(t_0) = -i\omega_k \frac{1 - f_k}{1 + f_k} \varphi_k(t_0)$$

$$\varphi_k = \frac{\varphi_k^0 + f_k \varphi_k^{0\,*}}{\sqrt{1 - f_k f_k^*}}$$

$$\varphi_k^0(t) = \frac{e^{-ik(t - t_0)}}{\sqrt{2\omega_k}}$$

An effective initial state—short-distance structure

- **If we could solve for the "true vacuum" it might not be the same as our low energy idea of the vacuum; an effective state parameterizes this difference**
  - **non-localities? non-commutative space-time? strongly interacting gravity?**

- **To our "vacuum" state this difference appears as new short-distance structure**

- **The propagator should also be consistent with our initial condition**
  - **this condition results in an extra term in the propagator associated with the structure of the initial state**

- **For an general initial state, a loop will also introduce sums of over the short-distance structure of the state**
  - **new divergences require *boundary counterterms***

for vacuum, 0

$$f_k = \text{"IR important"} + \text{"UV important"}$$

possible for *k* > *M*

$$f_k = \sum_{n=1}^{\infty} c_n \frac{k^n}{M^n}$$

UV important features of the state

⇓⇓

irrelevant counterterms on the initial boundary

A brief overview of the initial state renormalization

- **What emerges is an effective theory with many familiar properties**
  - **the long distance features are fixed empirically and any divergences are cancelled by *relevant* or *marginal* counterterms with respect to the boundary action**
  - **we include a general set of short distance features consistent with the symmetries of the state; their divergences also require *irrelevant* boundary counterterms**

hep-th/0501158, hep-th/0507081

- **Note that when regulating the theory, there is a single cutoff so both "bulk" and "boundary" counterterms depend on a single renormalization scale μ**
  - **Callan-Symanzik equation**
- **An effective theory of a state provides a model-independent description of the trans-Planckian effects**
  - **typical effect scales as *H/M***

Spergel (ISCAP, 5/2005)

- **But can such effects be seen?**
  - **CMB precision measurements (WMAP/Planck): 10^{-3}**
  - **LSS/galaxy surveys (Square kilometre array, …): 10^{-5}**
  - **note that we should include other subleading effects too, so it is important to determine both the *amplitude* (*H/M*) and the *shape* of the effective initial state signal**

Renormalization of a state and its evolution

- **Let us summarize what we have found, both for a flat and a completely general Robertson-Walker background,**

	IR/long distance			UV/short distance		
	structure	renormalization		structure	renormalization	
		operators	examples		operators	examples
bulk (evolution)	**observed long distance degrees of freedom**	**relevant, marginal (dim ≤ 4)**	$\nabla_\mu\varphi\nabla^\mu\varphi$, φ^2, φ^4, $R\varphi^2$	**completely free, up to assumed symmetries of background**	**irrelevant (dim > 4)**	. $\nabla_\mu\varphi\nabla^\mu\varphi^p$, φ^6, φ^8, $R^2\varphi^2$, …
boundary (state)	**appropriate state of long distance effective free theory**	**relevant, marginal (dim ≤ 3)**	φ^2, $\varphi\nabla\ \ \varphi$, φ^2	**completely free, up to assumed symmetries of state**	**irrelevant (dim > 3)**	φ^4, $(\nabla\ \ \varphi$□, $\nabla\ \ \varphi\nabla\ \ \varphi$ $^2\varphi^2$, …

- **Here, $\nabla_n = n^\mu\nabla_\mu$ is a derivative normal to the initial surface and $K_{\mu\nu} = h_\mu^{\lambda}\nabla_\lambda n_\nu$ is the extrinsic curvature along the surface**

Further work

- **So we find an elegant correspondence between the long and short distance features of the initial state and the sorts of operators the appear in their renormalization**

- **This is still rather a young subject so there are many aspects which should be studied further**
  - **back-reaction (size of effect, types of operators that appear)**
  - **RG flow (de Sitter space?)**
  - **decoherence of quantum effects**
  - **generating effective states by integrating out excited heavy fields**
  - **calculation of the amplitude and the generic shape of the trans-Planckian correction to the power spectrum**
  - **. . .**

Back-reaction and naturalness:

- **Porrati, 2004–2005**
- **Greene, Schalm, Shiu, & van der Schaar, 2004–2005**

Somewhat related work on RG flows in de Sitter space:

- **Larsen & McNees, 2003–2004**

Fits to the CMB data:

- **Easther, Kinney & Peiris, 2004–2005**