Gravitational Waves and the Microwave Background

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KICP Inaugural Symposium, December 11, 2005

Outline

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Microwave Temperature and Polarization Anisotropies

Microwave background radiation is described by its temperature and polarization as a function of sky direction: $T(\hat{\mathbf{n}})$ and $P_{ab}(\hat{\mathbf{n}})$

Temperature is a scalar function; polarization is a symmetric, traceless, rank-2 tensor function (2 degrees of freedom)

Common polarization Stokes Parameters Q and U depend on tensor basis

Harmonic Decompositions

Temperature:

$$T(\hat{\mathbf{n}}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} a_{(lm)} Y_{(lm)}(\hat{\mathbf{n}})$$

Polarization:

$$P_{ab}(\hat{\mathbf{n}}) = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} \left[a^{\mathrm{G}}_{(lm)} Y^{\mathrm{G}}_{(lm)ab}(\hat{\mathbf{n}}) + a^{\mathrm{C}}_{(lm)} Y^{\mathrm{C}}_{(lm)ab}(\hat{\mathbf{n}}) \right]$$

Polarization has two degrees of freedom, so two sets of basis functions are required to span the space of polarization fields

Tensor Spherical Harmonics

One convenient basis is the "gradient/curl" basis (Kamionkowski, Kosowsky, Stebbins 1997):

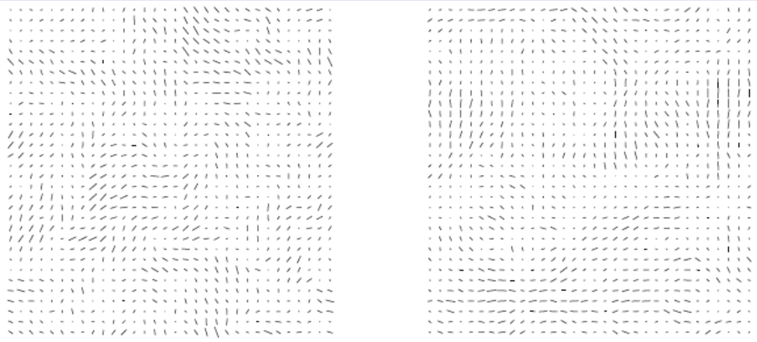
$$Y^{\rm G}_{(lm)ab} = N_l \left[Y_{(lm):ab} + \frac{1}{2} g_{ab} Y_{(lm):c}{}^{:c} \right]$$

$$Y^{\rm C}_{(lm)ab} = \frac{N_l}{2} \left[Y_{(lm):ac}\epsilon^c{}_b + Y_{(lm):bc}\epsilon^c{}_a \right]$$

where g_{ab} and ϵ_{ab} are the metric and antisymmetric tensors on the sphere, and ":" indicates a covariant derivative

This looks like the familiar gradient/curl decomposition of a vector field. Independent of polarization basis

G and C Polarization Modes



Random C (left) and G (right) polarization modes (Bunn 2003)

Geometry of Scalar and Tensor Modes

A physically useful basis:

One Fourier mode of a scalar perturbation has spatial dependence $e^{i\mathbf{k}\cdot\mathbf{x}}$: for fixed $\mathbf{k}$ depends only on angle between $\mathbf{k}$ and $\mathbf{x}$, so is axially symmetric. Therefore it cannot have any curl component: $a^{\mathrm{C}}_{(lm)} \equiv 0$ for scalar perturbations!

One Fourier mode of a tensor perturbation (a spin-2 field) has axial spatial dependence like $\cos(2\phi)$ so $a^{\mathrm{C}}_{(lm)}$ is nonzero in general.

Curl Polarization

Curl polarization of the microwave background radiation provides a means for detecting non-scalar primordial perturbations, namely primordial gravity waves (tensor perturbations) which are generated by inflation

Mathematically equivalent (except for normalization) to E-B basis of spin-2 spherical harmonics used by Seljak and Zaldarriaga (1997)

Real-World Effects

Unique E-B decomposition only on the full sky. On regions with boundaries, some modes are ambiguous! (Bunn et al. 2003; also Lewis, Challinor, and Turok 2002)

In all models, B-modes have a much smaller amplitude than E-modes, making their extraction technically challenging: see talk by C. Armitage; also K. Smith 2005

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Tensor Perturbations and Microwave Polarization

Gravitational Waves from Inflation

Detection Prospects

Inflation Basics

Inflation is a period in the early universe of accelerating expansion, which means that the stress-energy tensor must satisfy $w \equiv p/\rho < -1/3$.

The universe must undergo enough inflation to increase the scale factor by e^{60}, or 60 e-foldings. Then inflation must end.

Scalar density perturbations generated by inflation 60 e-foldings before the end must have amplitude of 10^{-5} to match fluctuations observed in the microwave background.

Fundamental Relations

These conditions imply that:

Energy scale of inflation $M \equiv \rho^{1/4} \approx 10^{-5/2}(1+w)^{1/4} m_{\rm Pl}$

Tensor-scalar ratio $\mathcal{R} \equiv (T/S)^2 \approx \rho/(\rho + p) \approx (1+w)$

To determine the tensor amplitude on current horizon scales, we must estimate $1+w$ at 60 e-foldings before the end of inflation.

The Simplest Inflation Models

The Hubble parameter $H \equiv \dot{a}/a$ must change by an amount of order unity between 60 and 0 e-foldings for inflation to end. Natural scale for $\dot{H}$ at a time Δt before the end of inflation is

$$\dot{H} \approx \frac{H}{\Delta t}$$

This implies that N e-foldings before the end of inflation, $1 + w \approx 1/N$. At $N = 60$, $1 + w \approx 1/60$, so

$$\mathcal{R} \approx 0.02 \qquad\qquad M \approx 10^{-3} m_{\rm Pl}$$

The Tensor-Scalar Ratio

The simplest inflation models give an energy scale for inflation at roughly the SUSY-GUT scale 10^{16} GeV, and with a substantial tensor contribution to the large-angle microwave background polarization fluctuations

Models with significantly smaller $\mathcal{R}$ can be shown to require two distinct mass scales $m_1 = V/V'$, $m_2 = V'/V''$ in the inflaton potential, satisfying $m_1/m_2 \approx N\mathcal{R}$ (argument due to P. Steinhardt)

(Also see L. Boyle presentation)

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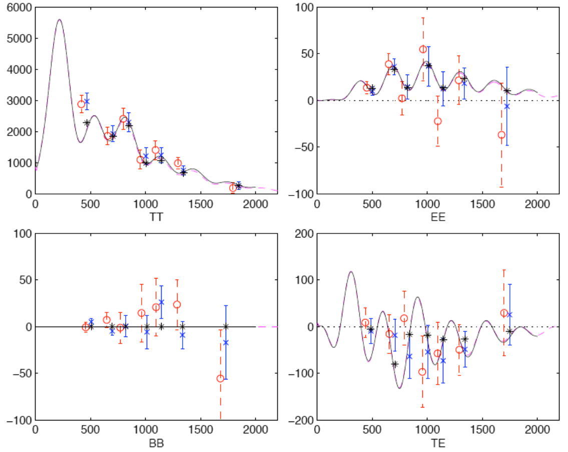
Current Polarization Detections



CBI: Sievers et al. (2005).

Also DASI, Boomerang, CAPMAP, WMAP detections

WMAP and Planck Limits

WMAP will be able to detect a tensor-scalar ratio of around $\mathcal{R} \approx 0.02$: barely sensitive to most optimistic inflationary model

Planck may be able to detect a tensor-scalar ratio of around $\mathcal{R} \approx 5 \times 10^{-4}$: can probe models with mass scale ratio of 0.03

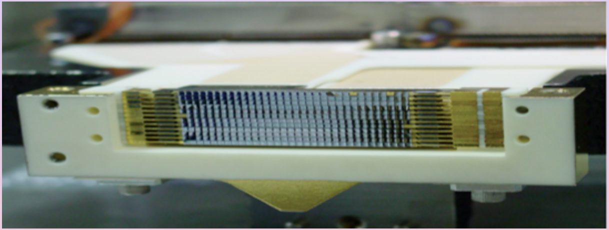
Future Space Missions

A dedicated polarization satellite is part of NASA's Beyond Einstein Program, which might probe $\mathcal{R} \approx 10^{-6}$. Also ESA SAMPAN concept study. (2015??) Can probe models with mass ratio of 10^{-4}

New technology: Large bolometer arrays with thousands of elements (ACT, SPT, SCUBA-2) will greatly increase sensitivity

See S. Church presentation; J. Fowler presentation

A Bolometer Array



The Sharc-II detector, 12×32 bolometer array (C.D. Dowell et al. 2002)

Outlook

Within the next decade, we can detect gravitational waves via large-angle CMB polarization measurements if the simplest models of inflation are correct.

Alternatives: (1) more complex and less elegant inflation models involving multiple energy scales; (2) inflation did not occur (cf. cyclic universe model of Steinhardt, Turok and collaborators)