Niew Views of the Universe, KICP Symposium   10th December 2005

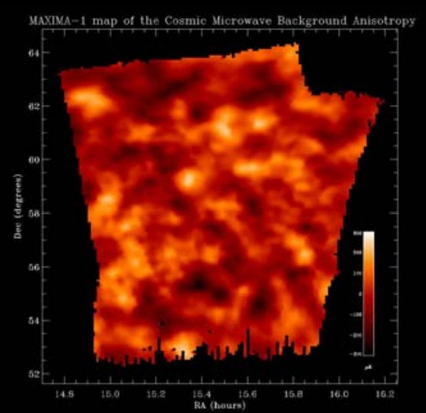
Inflation
and the origin of structure

David Wands
Institute of Cosmology and Gravitation
University of Portsmouth

Standard model of structure formation

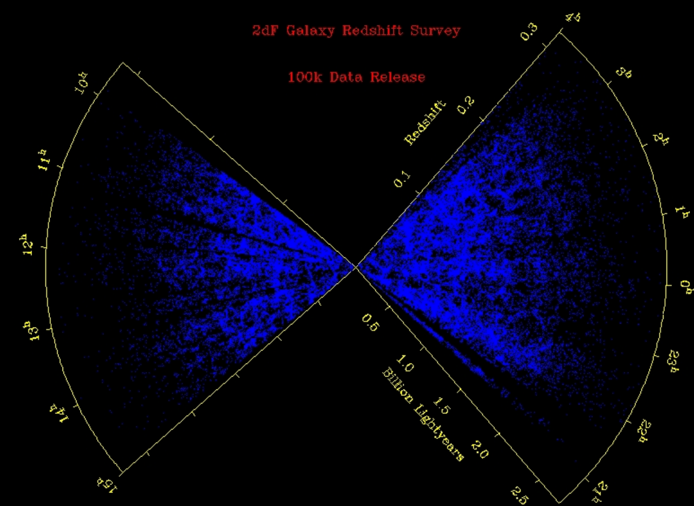
primordial perturbations

in cosmic microwave background



gravitational

instability



large-scale structure of our Universe

new observational data offers precision tests of

- **cosmological parameters**
- **the nature of the primordial perturbations**

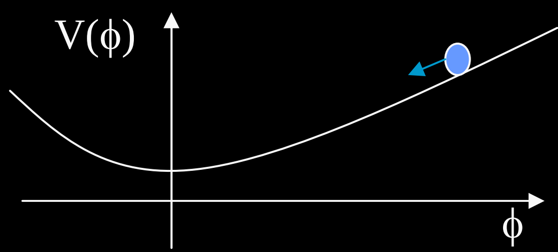
Inflation: initial false vacuum state drives accelerated expansion
zero-point fluctuations yield spectrum of perturbations

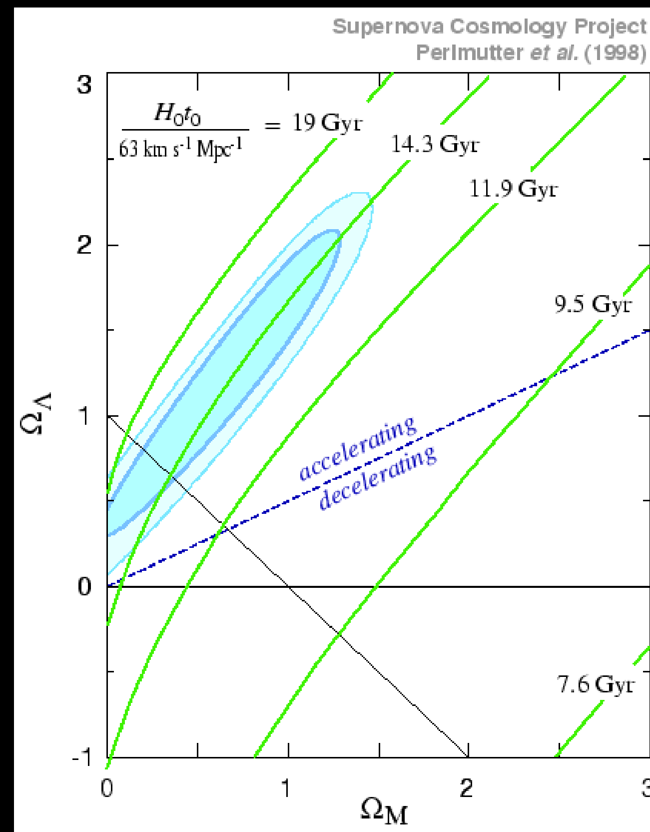
Cosmological inflation:

Starobinsky (1980)
Guth (1981)

- period of accelerated expansion in the very early universe
- requires negative pressure

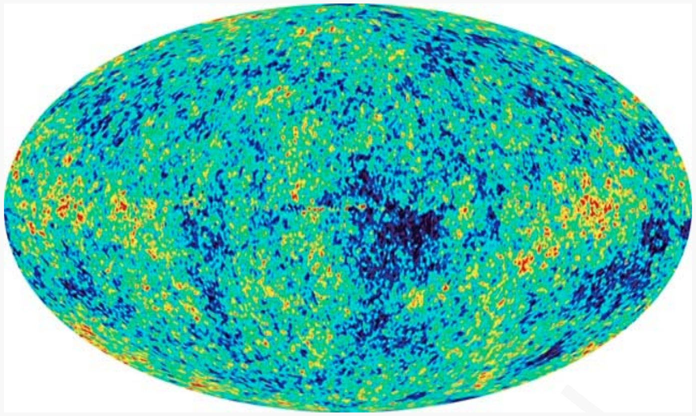
  e.g. self-interacting scalar field



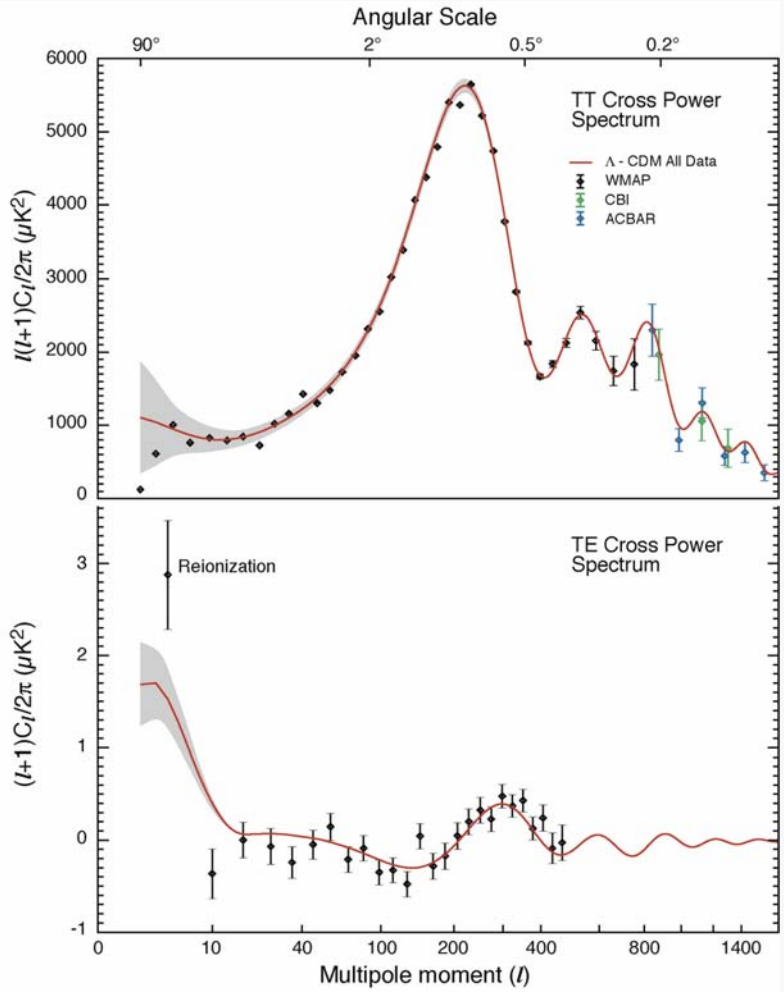


- speculative and uncertain physics
- just the kind of peculiar cosmological behaviour we observe today

Wilkinson Microwave Anisotropy Probe February 2003



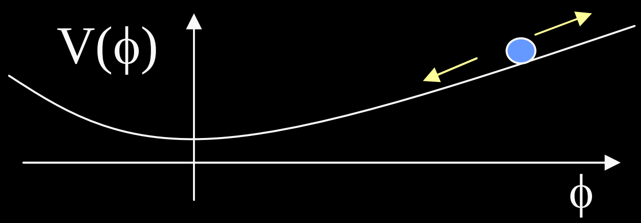
coherent oscillations
in photon-baryon plasma
due to primordial density perturbations
on super-horizon scales



linking the very small to the very large!

vacuum fluctuations
swept up by accelerated expansion

Hawking '82, Starobinsky '82, Guth & Pi '82



- *small-scale/underdamped zero-point fluctuations* $\delta\phi_k \approx \frac{e^{-ik\eta}}{a\sqrt{2k}}$
- *large-scale/overdamped perturbations in growing mode*
  linear evolution $\Rightarrow$ *Gaussian random field*

$$\left\langle \delta\phi^2 \right\rangle_{k=aH} \approx \frac{4\pi k^3 \left|\delta\phi_k{}^2\right|}{(2\pi)^3} = \left(\frac{H}{2\pi}\right)^2$$

fluctuations of any scalar light fields (m<3H/2) `frozen-in' on large scales

inflation probes high energies

- *cosmic expansion on large scales*

$$H^2 = \frac{8\pi}{M_{Pl}^2} V(\phi) + \dots$$

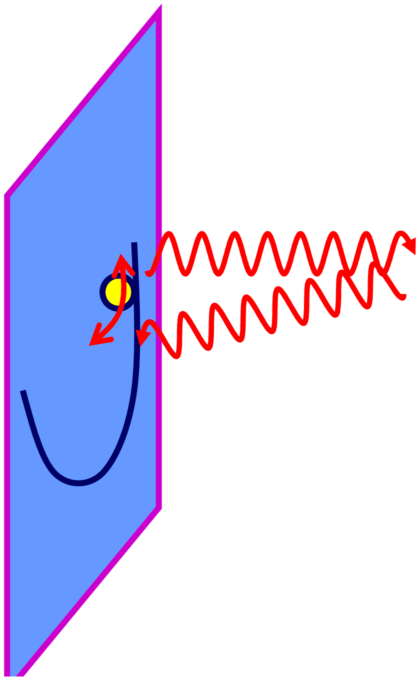
  - *reconstruct inflaton potential*
  - *modified Friedmann equation*

- *quantum vacuum on small scales*

$$\left\langle \delta\phi^2 \right\rangle = \left(\frac{k}{2\pi a} \right)^2 + \dots$$

  - *trans-Planckian effects (modified dispersion relation, Lorentz-violation...)*

Field perturbations on a brane coupled to metric perturbations



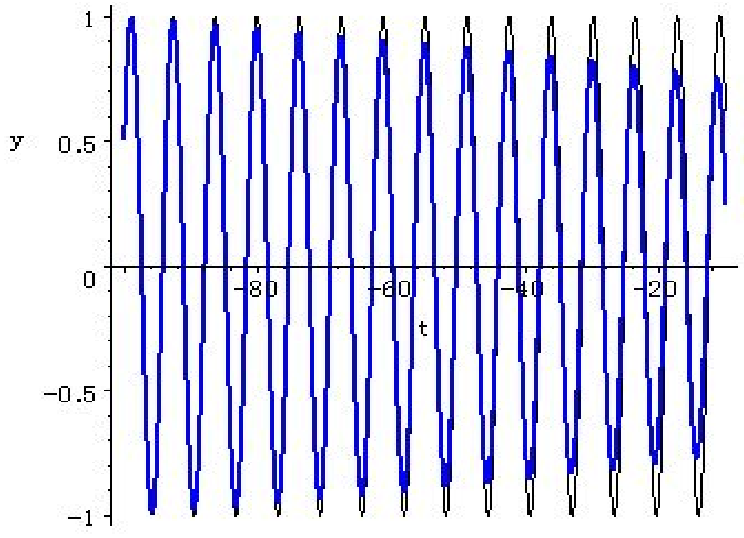
recover 4D gravity at low energies

but probe 5D at high energies

- 5D backreaction at high energy

can damp small scale oscillations

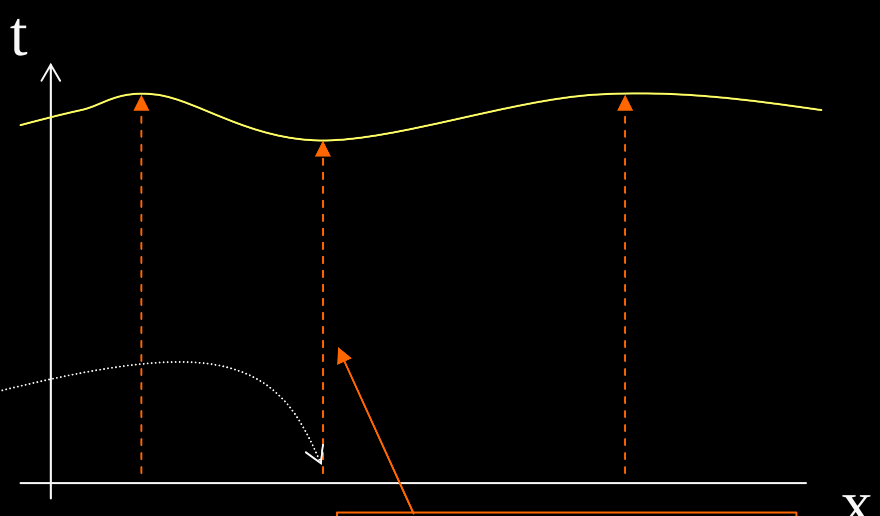
Koyama, Mizuno & Wands '05



advertisement: see talk by Andy Mennim on Monday!

primordial perturbations from scalar fields

in radiation-dominated era
    curvature perturbation ζ on
    uniform-density hypersurface



during inflation
    field perturbations $\phi(x,t_i)$ on
    initial spatially-flat hypersurface

$$N = \int_{initial}^{final} H \, dt$$

on large scales, neglect spatial gradients, treat as “separate universes”

$$\zeta = N(\phi_{initial}) - \overline{N} \approx \sum_I \frac{\partial N}{\partial \phi_I} \delta\phi_I$$

Sasaki & Stewart ’96

density perturbations from inflaton field perturbations

- *quantum fluctuations on spatially flat (δN=0) hypersurfaces during inflation*

$$\zeta = \frac{dN}{d\sigma}\delta\sigma = \left(-\frac{H}{\dot{\sigma}}\delta\sigma \right)_{k=aH}$$

- *produce density perturbations in radiation-dominated era*

$$\Rightarrow \quad \left\langle \frac{\delta T^2}{T^2} \right\rangle_{SW} \approx \frac{1}{25}\left\langle \zeta^2 \right\rangle \approx \frac{1}{25}\left(\frac{H^2}{2\pi\dot{\sigma}} \right)^2_{k=aH}$$

$$\text{tilt}: \quad n_\zeta - 1 \equiv \frac{d\ln\left\langle \zeta^2 \right\rangle}{d\ln k} \approx -6\varepsilon + 2\eta_\sigma \quad << \quad 1$$

$$\text{where} \quad \varepsilon = -\dot{H}/H^2 \quad , \quad \eta_\sigma = m_\sigma^2/3H^2$$

tensor metric perturbations

- *transverse, traceless metric perturbations*

$$\delta g_{ij}(t,x) \approx \int d^3k\; h_k(t)\, e_{ij}^{(+,\times)}(x)$$

- *amplitude,* h(t), *obeys same wave equation for massless field*

- *remain decoupled from matter perturbations*

$$\Rightarrow \quad \left\langle T^2 \right\rangle \;\approx\; \left(\frac{64\pi}{M_{Pl}^{\;2}} \right) \left(\frac{H}{2\pi} \right)^2_{k=aH}$$

$$\text{tilt}: \quad n_T \;\equiv\; \frac{d \ln \left\langle T^2 \right\rangle}{d \ln k} \approx \; -2\varepsilon \qquad \text{where} \quad \varepsilon \equiv -\dot{H}\big/H^2$$

"smoking gun" for inflation...

- *inflation predicts primordial gravitational wave background*

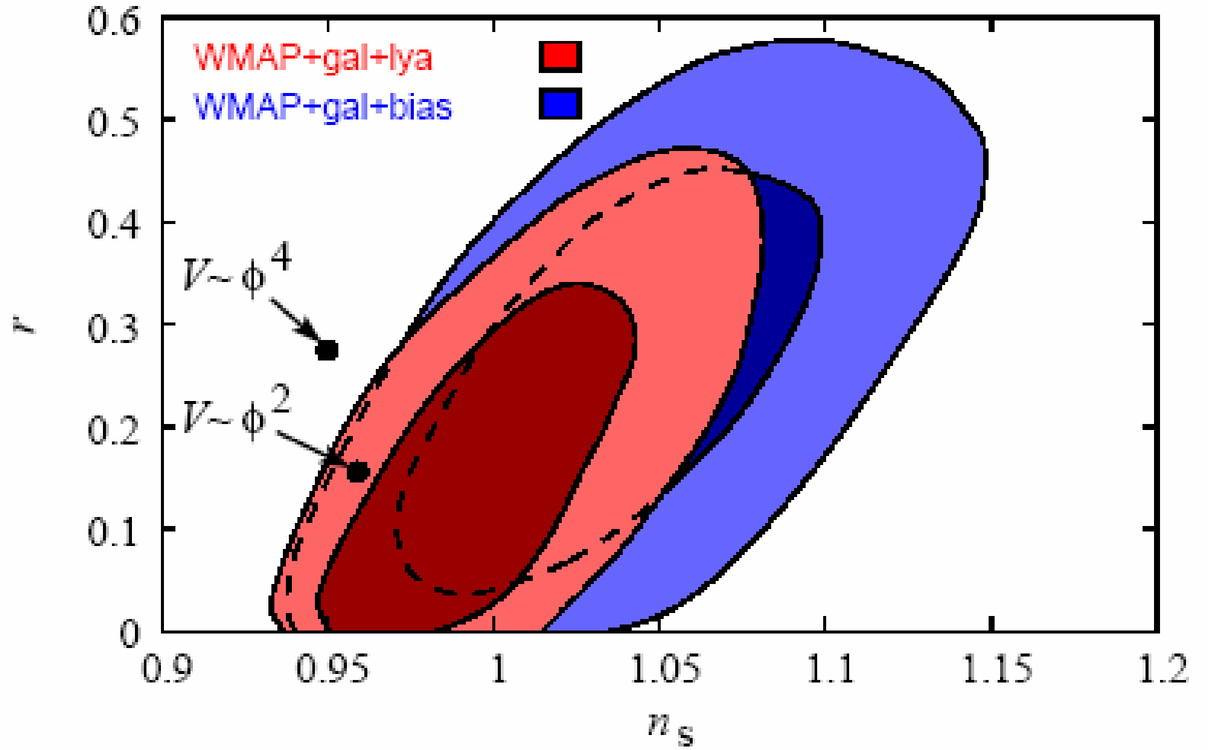
$$\langle T^2 \rangle \approx \left(\frac{V}{M_{Pl}{}^4} \right)_{k=aH}$$

- *could be* $$\left(\frac{10^{16} GeV}{M_{Pl}} \right)^4 \approx 10^{-12}$$

- *or could be* $$\left(\frac{1 TeV}{M_{Pl}} \right)^4 \approx 10^{-64}$$

- *only detectable if inflationary scale > 10^{15} GeV*

$$r = \frac{\langle T^2 \rangle}{\langle \zeta^2 \rangle} \approx 16\varepsilon$$



Seljak et al (2004)

but fluctuations in other fields can also perturb radiation density after inflation

- ***coupled fields during slow-roll during inflation***

  Starobinski & Yokoyama; Sasaki & Stewart; Mukhanov & Steinhardt; Linde, Garcia-Bellido & Wands…. (1995)

- ***curvaton decay after inflation***

  weakly-coupled, late-decaying scalar field

  Enqvist & Sloth; Lyth & Wands; Moroi & Takahashi (2001)

- ***inhomogeneous / modulated reheating or preheating***

  inflaton decay-rate modulated by another light field

  Dvali, Gruzinov & Zaldariaga; Kofman (2003); Kolb, Riotto & Vallinotto (2004)

- ***inhomogeneous end of inflation***

  Lyth; Salem (2005)

primordial perturbations from isocurvature fields during inflation

- *quantum fluctuations on spatially flat ($\delta N=0$) hypersurfaces during inflation*

$$\zeta = \frac{dN}{d\chi}\delta\chi$$

where N(χ) dependent on physics

- *produce density perturbations in radiation-dominated era*

$$\Rightarrow \left\langle \frac{\delta T^2}{T^2} \right\rangle_{SW} \approx \frac{1}{25}\left\langle \zeta^2 \right\rangle \approx \frac{1}{25}\left(\frac{dN}{d\chi}\right)\left(\frac{H}{2\pi}\right)^2_{k=aH}$$

  - *amplitude depends upon physics*
  - *but spectral tilt set during inflation*

$$\text{tilt}: \quad n_\zeta - 1 \approx -2\varepsilon + 2\eta_\chi$$

$$\text{where} \quad \varepsilon = -\dot{H}/H^2 \quad , \quad \eta_\chi = m_\chi^{\ 2}/3H^2$$

chaotic inflation:

$$V = \frac{1}{2} m^2 |\phi|^2 = \frac{1}{2} m^2 \left(\sigma^2 + \chi^2 \right)$$

$$\varepsilon = \eta_{\sigma\sigma} = \eta_{\chi\chi} \approx 0.01$$

(I) inflaton perturbations

$$n = 1 - 6\varepsilon + 2\eta \approx 0.96$$

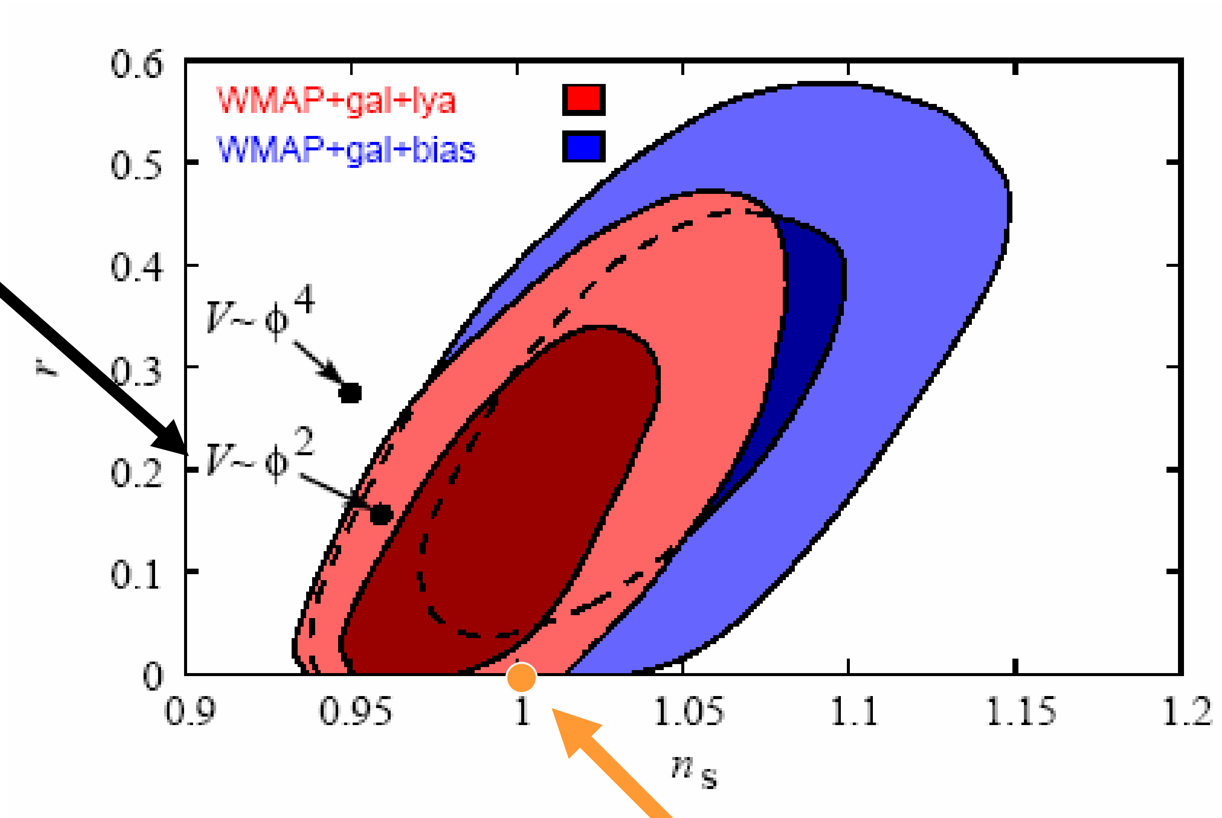
$$r \approx 0.16$$

(II) isocurvature perturbations

$$n = 1 - 2\varepsilon + 2\eta \approx 1$$

$$r << 0.16$$

inflaton



Seljak et al (2004)

modulated preheating?

Byrnes & Wands (astro-ph)

distinctive observational predictions

- inflaton perturbations
  - adiabatic
  - no isocurvature perturbations
  - Gaussian

- isocurvature field perturbations
  - non-adiabatic
  - *possible* residual isocurvature modes…
  - … correlated with curvature perturbation
  - *possible* non-Gaussianity

linear evolution -> Gaussian perturbations stay Gaussian

non-linear evolution -> non-Gaussianity!

single-field inflation	Maldacena (2002);
	Acquaviva, Bartolo, Matarrese & Riotto (2002+);
beyond slow roll	Creminelli & Zaldarriaga (2003);
	Lidsey & Seery (2004);
multi-field inflation	Rigopoulos, Shellard & van Tent (2003+)
	Lyth & Rodriguez (2004);
	Allen, Gupta & Wands (2005)

Gaussian field perturbations to first order

$$\zeta_1 = \left(\frac{\partial N}{\partial \phi} \right) \delta_1 \phi$$

give non-Gaussian metric perturbation to second order

$$\zeta_2 = \left(\frac{\partial N}{\partial \phi} \right) \delta_2 \phi + \frac{1}{2} \left(\frac{\partial^2 N}{\partial \phi^2} \right) (\delta_1 \phi)^2$$

“local” non-Gaussianity

simplest kind of non-Gaussianity:

Komatsu & Spergel (2001)

Wang & Kamiokowski (2000)

$$\zeta = \zeta_1 - \frac{3}{5} f_{NL} \zeta_1^{\,2}$$

gives bispectrum:

$$B_\zeta(k_1, k_2, k_3) \propto f_{NL}\left[P(k_1)P(k_2) + P(k_2)P(k_3) + P(k_3)P(k_1)\right]$$

constraints from WMAP: -58 < f_{NL} < 134

more data to come...

Detectable non-Gaussianity can come from non-adiabatic perturbations in non-inflaton fields

Lyth & Rodriguez (2005)

- perturbation due to inflaton field

$$\zeta_1 = N_{,\sigma}\delta_1\sigma$$

$$\zeta_2 = N_{,\sigma}\delta_2\sigma + N_{,\sigma\sigma}\left(\delta_1\sigma\right)^2$$

- where $N_{,\sigma} = H/\dot{\sigma}, \quad N_{,\sigma\sigma} = -(\dot{H}/H^2) + (\ddot{\sigma}/H\dot{\sigma})$ can be calculated during inflation
- $N_{,\sigma\sigma}$ must be *small* during slow-roll inflation
- *but* perturbations from non-adiabatic perturbations dependent upon subsequent expansion history

$$\zeta_1 = N_{,\chi}\,\delta_1\chi$$

$$\zeta_2 = N_{,\chi}\delta_2\chi + N_{,\chi\chi}\left(\delta_1\chi\right)^2$$

non-Gaussianity from curvaton decay

simplest kind of non-Gaussianity:

$$\zeta \approx \zeta_1 - (3/5) f_{NL} \zeta_1^2$$

Komatsu & Spergel (2001)

Wang & Kamiokowski (2000)

recall that for curvaton

$$\zeta \approx \Omega_{\chi,\mathrm{decay}}\, \zeta_\chi \approx \Omega_{\chi,\mathrm{decay}} \left(\frac{\delta\chi}{\chi} + \left(\frac{\delta\chi}{\chi} \right)^2 \right)$$

corresponds to

$$\zeta_1 \approx \Omega_{\chi,\mathrm{decay}} \left(\frac{\delta\chi}{\chi} \right) \quad , \quad f_{NL} \approx -\frac{5}{6\,\Omega_{\chi,\mathrm{decay}}}$$

Lyth, Ungarelli & Wands '02

constraints on f_{NL} from WMAP - $f_{NL} < 58$

hence $\Omega_{\chi,\mathrm{decay}} > 0.01$ and $10^{-5} < \delta\chi/\chi < 10^{-3}$

Conclusions:

- ***Inflation*** *links very small scale* ***vacuum fluctuations*** *to very large scale* ***structure of our universe***
- ***Precision cosmology*** (especially cosmic microwave background data) offer detailed measurements of ***primordial density perturbations***
- ***Gravitational waves, primordial isocurvature*** perturbations and/or ***non-Gaussianity*** could provide valuable info about origin of perturbations
- ***Single-field slow-roll inflation*** predicts ***adiabatic density perturbations*** with ***negligible non-Gaussianity*** *could give* ***detectable gravitational waves***
- ***Multi-field inflation*** allows ***non-adiabatic perturbations during inflation,*** which could give ***detectable primordial isocurvature perturbations*** and/or ***local non-Gaussianity***