Relaxing to Three Dimensions

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Introduction

- Very basic question about the world is why we seem to experience three spatial dimensions
- Are three dimensions special?
- We'll address this question in a higher-dimensional universe
- Show that not only is it possible to mimic three dimensions, might even be likely!

Context

- Lots of attention of late paid to the landscape idea
- In some scenarios (string theory, inflation) there are many possible vacua
- We live in one of them
- That part of the story is very likely true

But

- However, often combined with anthropic principle
- Before making predictions based on known vacua
- Want to determine as many possibilities as we can
- Perhaps qualitatively new phenomena in vacua we haven’t yet discovered

So far

- The vacua most string theorists study are based on curled-up dimensions
- Essentially all such work assumes six dimensions compactified
- Even more restrictive: almost all assume Calabi-Yau compactification

Alternative selection principles possible

- Localization—where gravity is concentrated in a finite region even though extra dimensions are infinite-- can perhaps help exploit new selection principles for finding where we live
- After all, initial evolution would be quite different
- Want to ask –quite generally--could there be a dynamical explanation for more observable aspects of the universe?
- And does localized gravity help us understand such phenomena?

The Relaxation Principle

- Proposal: The Relaxation Principle
- The universe naturally evolves into the favored vacuum: the one with maximum filling fraction
- Cosmological evolution determines which of many possible vacua are favored
- Shinji Mukohyama and I applied this idea

(with partial success) to the cosmological constant problem

Localized gravity and the relaxation principle

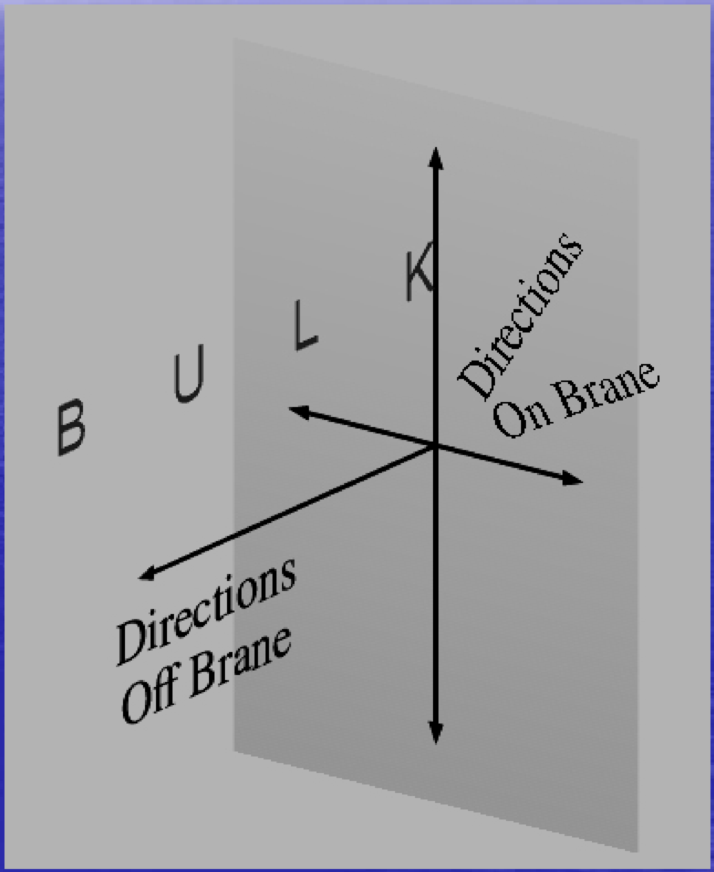
- Localized gravity gives a new scenario in which to apply the relaxation principle
- Potentially address difficult questions such as
  - Why are there three dimensions of space that we experience?
  - Why do forces act in three spatial dimensions?
  - Why should three dimensions be special at all??

Outline

- Review Localized Gravity
- Review old ideas for explaining 3 dimensions of space
- New ideas
- Potential relevance of localized gravity to explaining relevance of there spatial dimensions
- Conclude

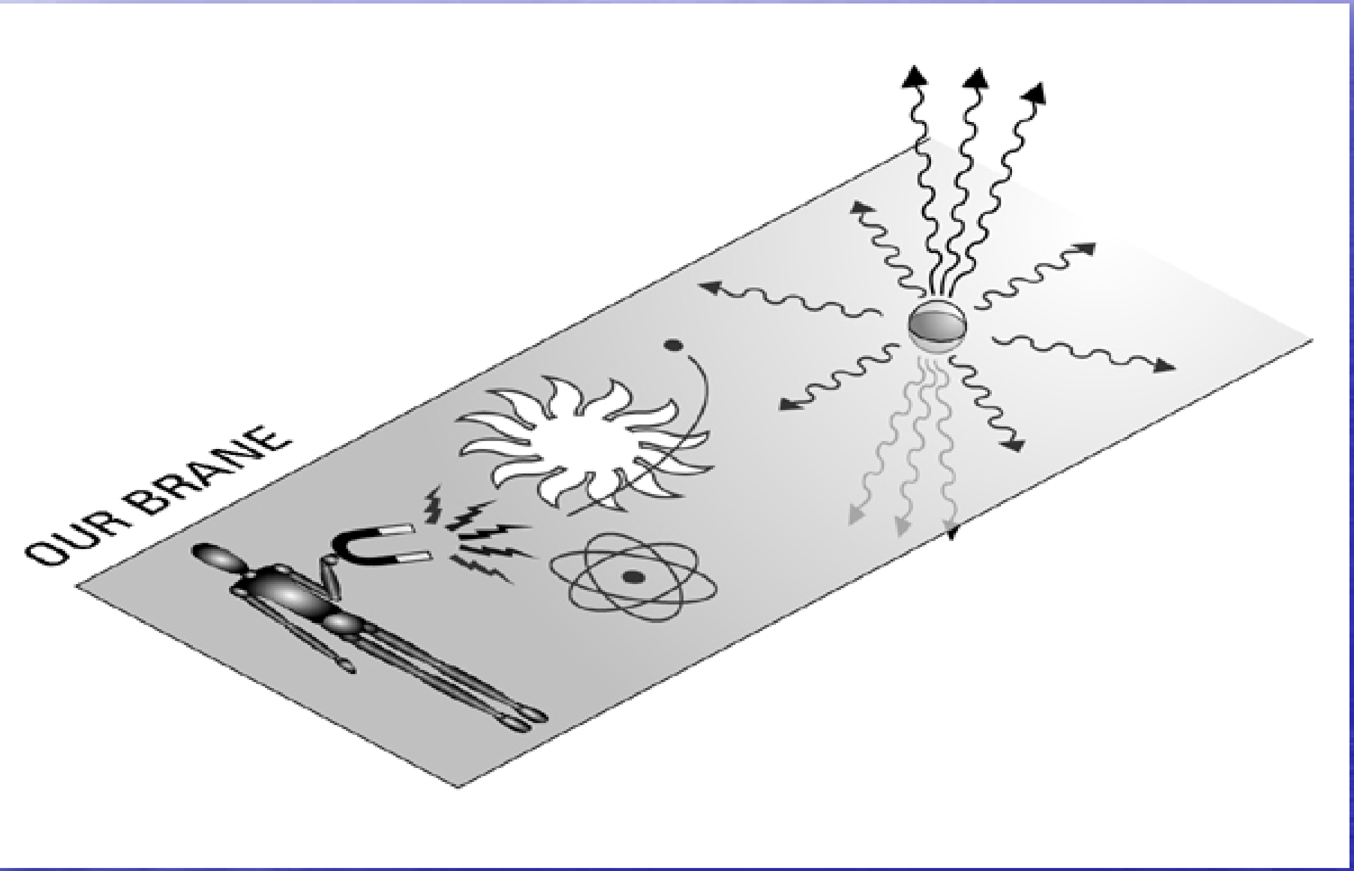
Catalyst for Recent Developments about extra dimensions: Branes

- Branes are membrane-like objects in higher-dimensional space
- Differentiate space on and orthogonal to brane
- Can confine particles and forces on their surface



Braneworld

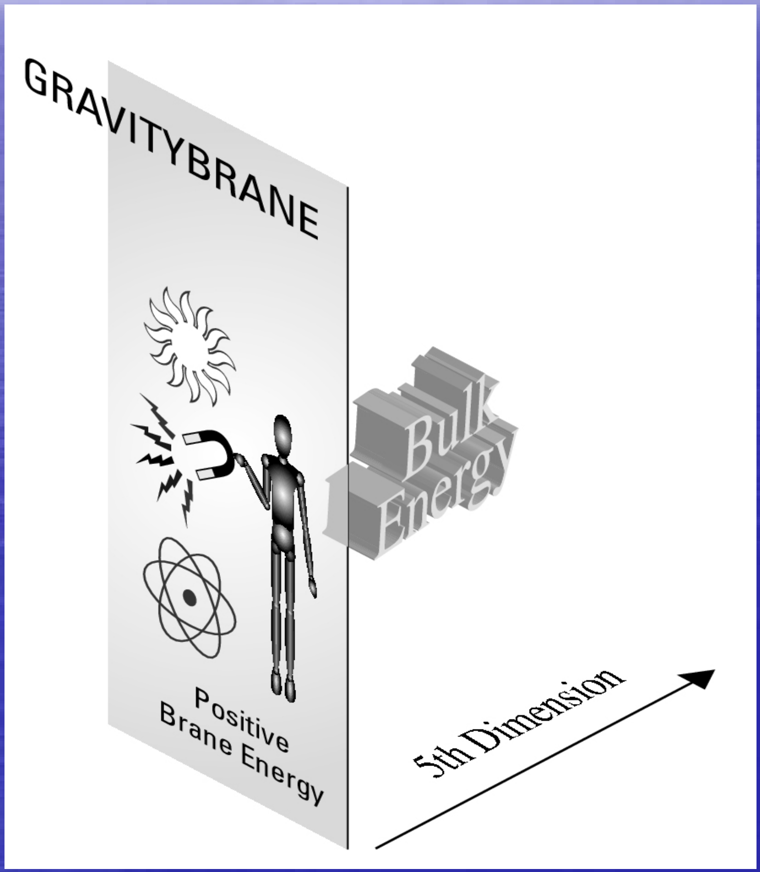
Particles can be confined to branes
Everything but gravity on a brane



Branes that Distort Space: New Way to Hide Dimensions

- If we're confined to branes, only problematic aspect of dimensions is gravity
- However, energetic branes in an energetic bulk can distort space
- So much so that an infinite extra dimension is possible
- Gravitational field (and graviton) get localized near a brane—RS2

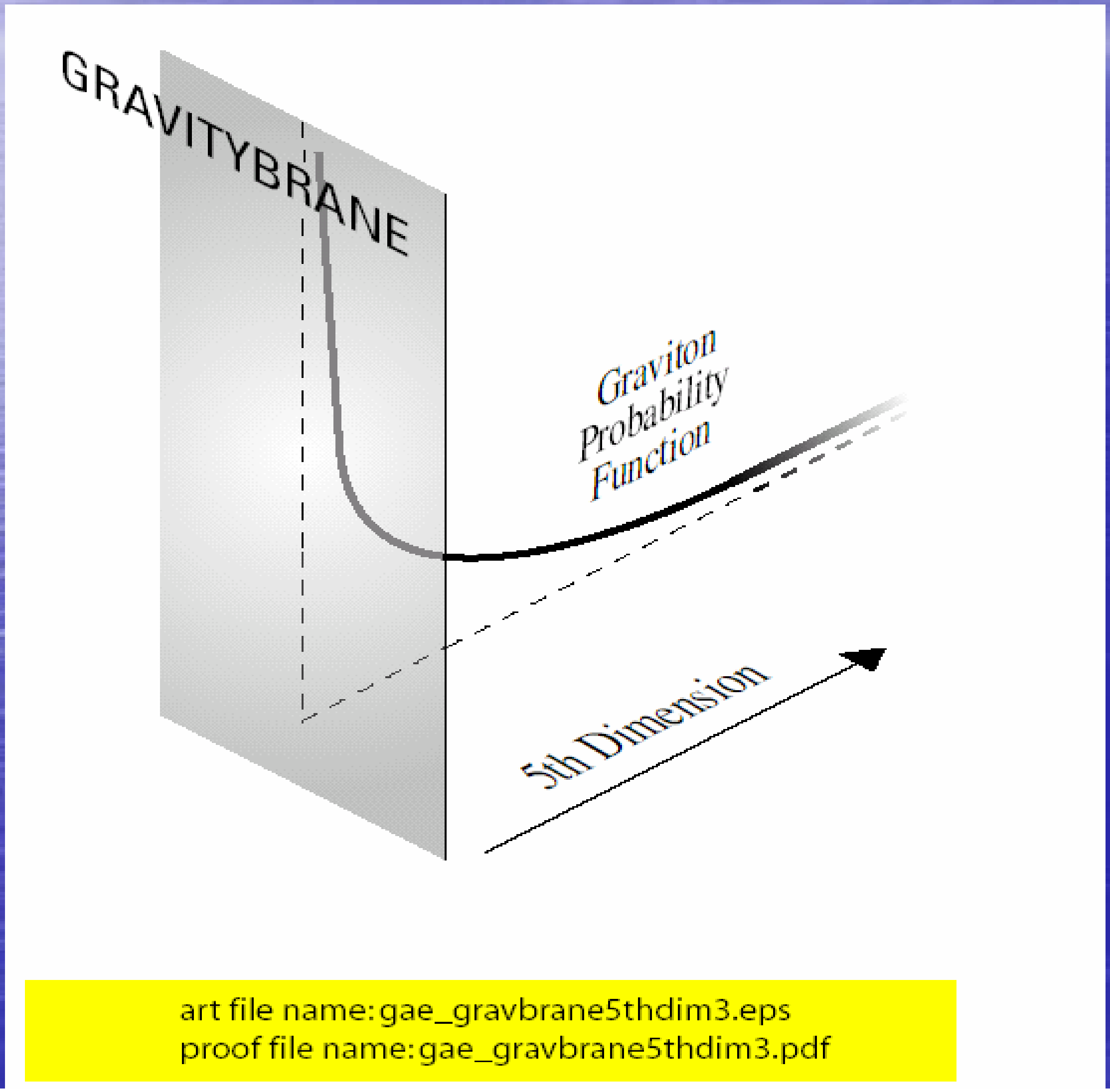
Scenario for Warped Geometry



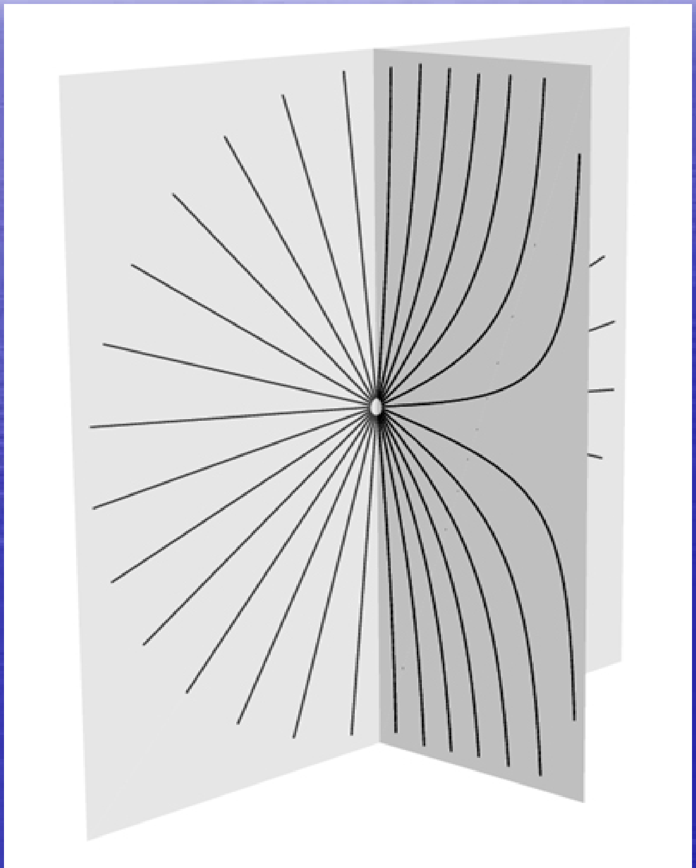
Find Localized gravity

- Lower-dimensional gravity survives in this warped space
- $ds^2 = dr^2 + e^{-kr}(dx^\mu \, dx^\nu \, \eta_{\mu\nu})$
- Warped metric: overall scale factor
- Here it exponentially decreases
- Consequence is that the zero mode in a KK reduction, e^{-kr}, is normalizable
- Find you get a four-dimensional graviton
- Even though space is fundamentally five-dimensional

Graviton in this geometry



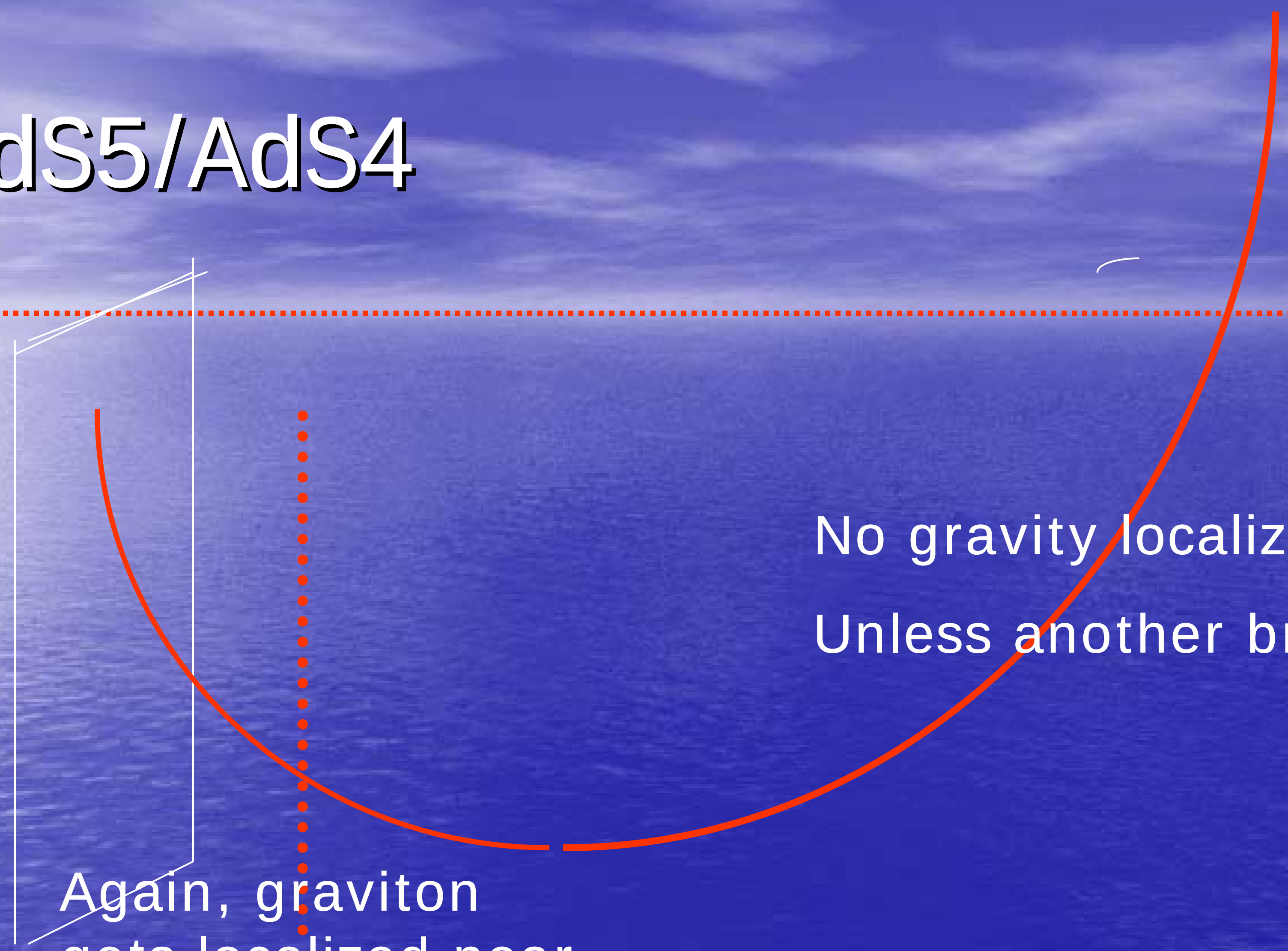
Gravitational Field Near Brane



Even More Dramatic: Locality of Four Dimensions

- Why should you need to know about space far from the brane?
- w/ Karch, an example based on AdS brane
- Warp factor turns around
- We find four-dimensional gravity (mediated by massive graviton) near the brane!

AdS5/AdS4



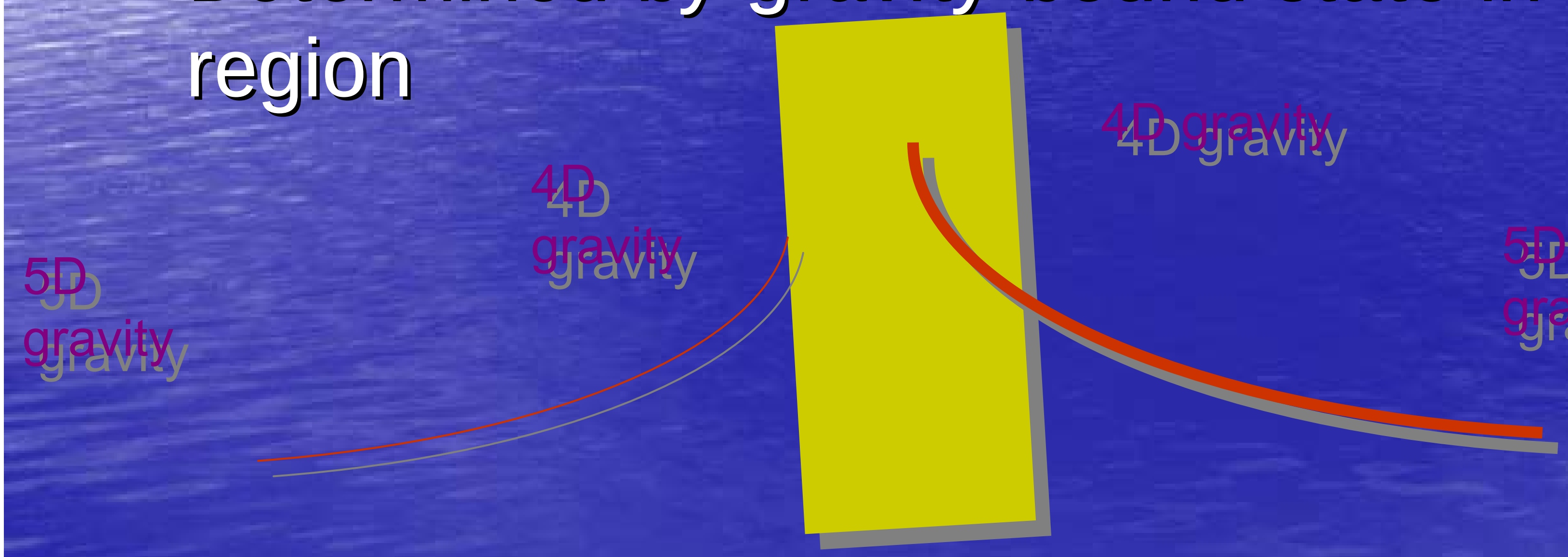
No gravity localization

Unless another brane

Again, graviton gets localized near brane

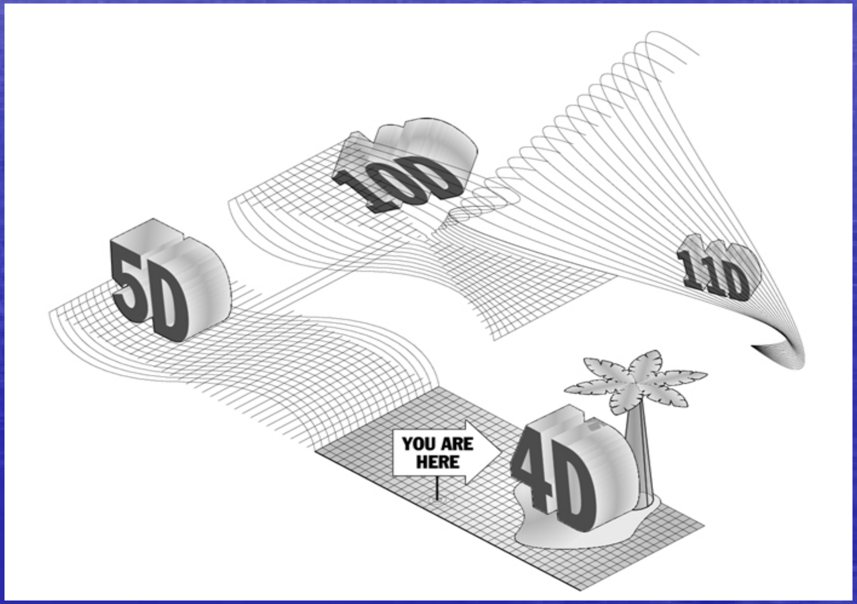
Locally localized gravity

- Dimensionality depends on location
- See different dimensions in different places
- Determined by gravity bound state in that region



Four dimensions might be a local phenomenon

- Why should you need to know about space far from the brane?
- four-dimensional gravity (mediated by massive graviton) near the brane
- But higher-dimensional elsewhere
- Copernican revolution continues!



- So it’s conceivable that space looks three-dimensional even though the world is truly higher-dimensional
- But is there a reason to single out three dimensions?
- We know three dimensions is possible but is it likely?

Existing Ideas to explain three spatial dimensions (in string theory)?

- Most popular: Brandenberger-Vafa proposal
- They assume compactified dimensions
- (As does everyone else who tries to ask this question)
- Their idea is that only a few dimensions grow large—
- Those for which there isn't an opposing force due to wrapped strings

Brandenberger-Vafa

- In general, there will be wrapped strings
- UNLESS the strings can meet and annihilate
- However, strings won't meet if there are more than four-dimensions in spacetime (2+2=4)
- But if there are four or fewer dimensions, the dimensions will grow larger
- Conclude: 4 (or fewer) dimensions will be large

Nice idea

- But...
- Depends on initial conditions
- Depends on poorly understood dynamics—everything is happening at the Planck scale
- Depends on moduli stabilization (what ultimately determines size and shape) and assumes simple toroidal compatification
- Neglects nonstring objects in “string”theory—branes

Four dimensions in context of branes and localized gravity
w/Andeas Karch

- Make opposite assumptions from BV
- Assume the number of large dimensions of space is fixed (by string theory?)
- Ask instead which branes (that is which dimensionalities of branes) survive
- If 3-branes survive, possible candidates for a four-dimensional universe
- If 3-branes have the biggest filling fraction, they are the most likely candidate

Basics

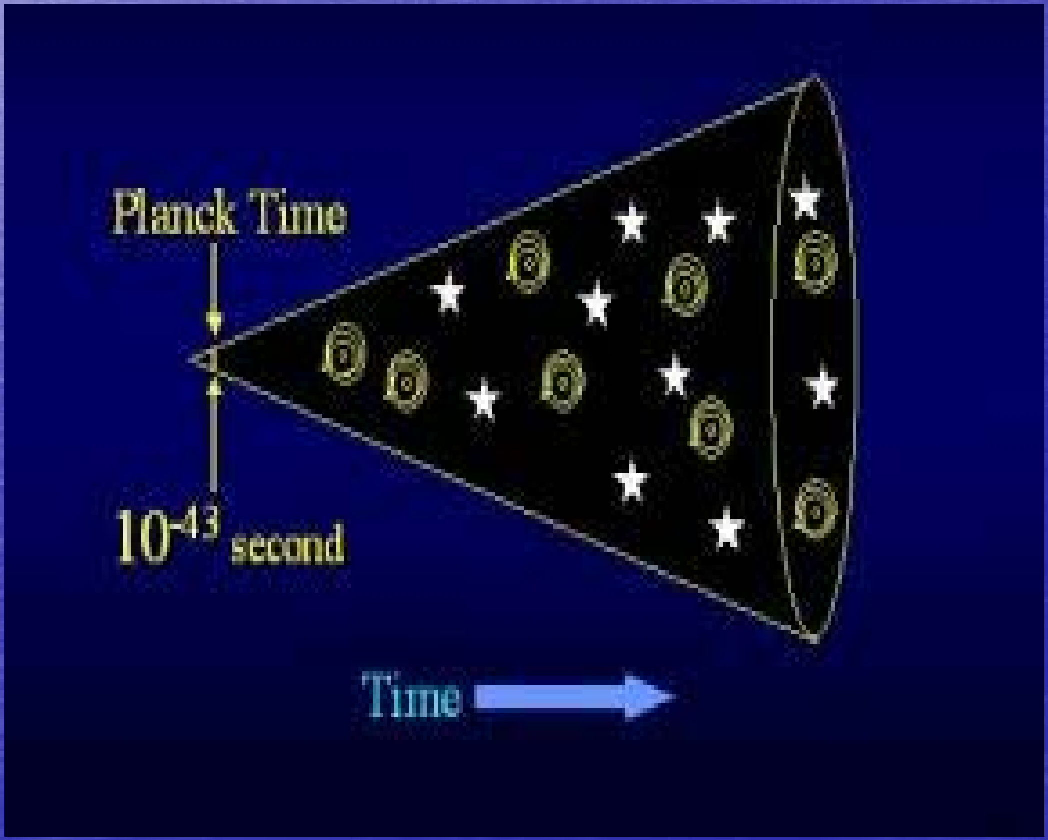
- Consider a ten-dimensional universe
- And let it evolve according to conventional FRW evolution
- Assume universe starts with an equal number of branes and antibranes (generic initial conditions; branes at random orientations)
- Of all dimensions
- Let the energy of the branes determine the equation of state that enters the FRW evolution

What happens?

- During the universe's evolution, some branes will dilute more than others
- After some time, we'll be left with a universe in which certain types of branes will be much more numerous
- Those are likely to be the ones on which we live

10 d FRW evolution

$$ds^2 = -dt^2 + a^2(t)\, d\,\Sigma_k^2$$



Evolution in n-dimensions with $p=w\rho$

$H^2=-k/a^2+16\pi G_N\rho/(n(n-1))$

With energy conservation

k

$$\frac{\dot\rho}{\rho} = -n(1+w)\frac{\dot a}{a}$$

s

$$\dot H = \frac{1_k}{a^2} - \frac{8\pi G_N}{(n-1)}(\rho+p)$$

More evolution

- Assume the equation of state is dominated by a single component (single w)
- Even if not initially true, will be the stable attractor of the evolution
- Solve

$$\rho \sim a^{-n(1+w)} \rightarrow t \sim a^{n/2(w+1)}$$

Equation of state for branes

- We are interested in a brane-dominated universe
- (We'll see that's what happens if we mix branes and dust and radiation in any case)
- Let's consider the argument for strings and generalize
- String has $\rho=-p$ like a cc, but only one component of p is nonzero
- So with strings in random directions in n=3, $p=-\rho/3$

Brane equation of state

- Using exactly the same logic
- For a d-brane with a d+1-dimensional world volume
- In n spatial dimensions
- $w=-d/n$
- From this, we conclude
- $a \sim t^{(n-d)/2}$

How do branes evolve in this expanding universe?

- If branes are non-interacting, we know how they scale
- Even without knowing the FRW evolution precisely
- The volume of the branes goes like a^d
- Whereas the volume of space goes like a^n
- So the density of d-branes (here d is merely dimension)
- a^{d-n}

- Clearly if this were all that were going on, the largest branes would dominate the energy density after enough time has elapsed
- But this is not the case
- In a 10-dimensional universe, 9-branes and anti-9-branes overlap everywhere
- So they will instantaneously annihilate
- And cannot dominate the energy density

8-branes

- 8-branes are tricky
- They always overlap over most coordinates and possibly annihilate
- Also in flat metric, 8-branes cannot be very far apart with space becoming singular
- Finally, if we have IIB and a complex tachyon, we don't expect 8-branes to form
- Upshot: 8-branes potentially a problem, but many reasons to think they aren't there
- (As with domain walls in four-dimensional universe)

Other branes

- Here's where things become interesting
- And we see that 3-branes are special
- Higher-dimensional branes must always intersect
- Their world volumes are such that
- $2(d+1) \geq 10$ ($=n+1$ for what we are interested in)
- So for us, rather than $2+2=4$, use $4+4<10$!

Consequences

- The density of branes with more than 3+1 dimensions is less than you would think
- We use the same (now standard) argument that is used for strings to establish their dilution rate
- Strings that intersect spawn loops
- Those loops radiate energy away through gravitational waves
- We'll assume similar dynamics for branes

Intersecting brane density

- Assume decay processes happen at the maximum efficiency allowed by causality
- The network at all times looks the same when viewed on the scale "t", the horizon size
- Scaling solution
- Length of string is some number times t
- Volume of d-brane some number times t^d

Cont'd

- Horizon volume is t^n so

$$\rho^i_d \sim t^{d-n}$$

True for any branes that can find each other and interact and trigger decay that's limited only by causality

Who Wins?

- Suppose 3-branes dominate
- w=-1/3 $t \sim a^{n/2(w+1)}$
- $a \sim t^{1/3;}$ 7-branes: $t^{-(9-7)}$; 3-branes: $a^{-(9-3)}$
- 7-branes and 3-branes dominate
- Density for both decreases as t^2
- All other branes dilute more quickly
- (Possible concern about 8-branes
- But no worse than domain walls for cosmic-string dominated scenarios)

Where are we?

- The only stable evolution has 3-branes and 7-branes dominating
- No other set of branes can consistently dominate the energy density
- Remarkable result
- 3-branes and 7-branes appear in particle physics and cosmology
- Used only generic assumptions and evolution

- 3-branes winning might seem best but
- 3-branes and 7-branes are both needed to have viable gauge theory with matter

- Furthermore, 3-branes and 7-branes can generate an inflationary scenario

Localized Gravity in Ten Dimensions w/ Liam Fitzpatrick

- System of 3 intersecting 7-branes
- (0,1,2,3,4,5,6,7,8,9)
- (x,x,x,x,x,x,x,x,-,-)
- (x,x,x,x,x,x,-,-,x,x,)
- (x,x,x,x,-,-,x,x,x,x)
- Generically the 3 branes intersect over a four-dimensional world volume
- Find gravity gets localized on the intersection
- New geometries and cosmological scenarios

Why is there gravity?

- We want to show that we have localized gravity at that intersection
- To do so, we combine (and generalize) two ideas
- Gherghetta, Shaposhnikov-codimension-2 localizaiotn
- Arkani-Hamed, Dimopoulos, Kaloper-intersecting codimension-1 branes

Gherghetta-Shaposhnikov set-up

- They assume a string-like defect in AdS6

$$ds^2 = \sigma(\rho) g_{\mu\nu} dx^\mu dx^\nu - d\rho^2 - \gamma(\rho) d\theta^2$$

$$T_\nu^\mu = \delta_\nu^\mu f_0(\rho), \quad T_\rho^\rho = f_\rho(\rho), \quad \text{and} \quad T_\theta^\theta = f_\theta(\rho)$$

Transverse Dimensions

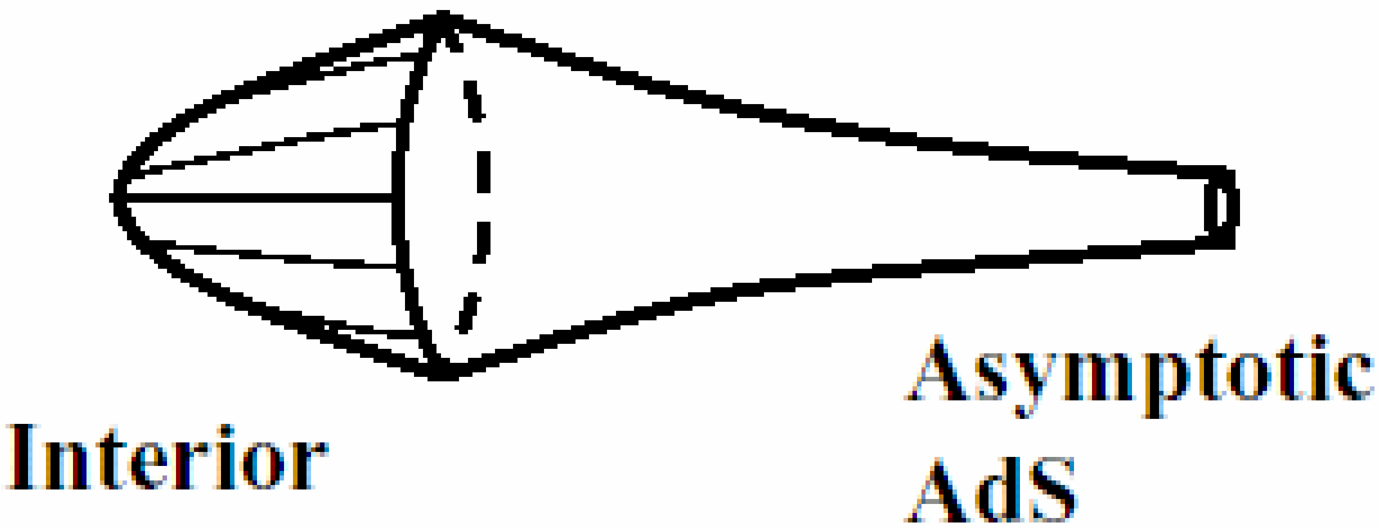


FIG. 1: Geometry of the transverse dimensions of the codimension 2 brane in asymptotically AdS space.

Intersecting codimension-1 branes

- Idea is that if we can get localization from a single codimension-1 brane
- Generating gravity on one lower dimensional volume
- We should be able to get (d+1)-dimensional gravity if we have n-(d+1) branes

Einstein's Equations

$$R^a{}_b - \frac{1}{2}\delta^a{}_b R = \kappa^2_{4+n}\Lambda\delta^a{}_b$$

$$- \frac{\sqrt{g_{3+n}(1)}}{\sqrt{g_{4+n}}}\kappa^2_{4+n}\sigma\delta(\bar{z}^1)\mathrm{diag}(1,1,1,1,0,...,1,1)$$

$$- ...$$

$$- \frac{\sqrt{g_{3+n}(n)}}{\sqrt{g_{4+n}}}\kappa^2_{4+n}\sigma\delta(\bar{z}^n)\mathrm{diag}(1,1,1,1,1,1,...,0),$$

Our Setup

- We combine these two ideas
- Matching is a little tricker because we have two nontrivial metric components
- And two nontrivial tensions
- Key is to treat the additional curled-up dimension as an additional flat dimension
- Take into account the fact that the metric components change at the brane
- Also show you can use Stoke's Theorem to get an exact result
- Also account for inner contribution from string-like singularity

Expect

- AdS outside (suitable warping) and therefore a normalizable zero mode
- Conditions on various tension components
- More complicated because not just delta-function singularities
- Nonetheless, gravity localized at the intersection

Result

Get 3-d gravity

- 3 branes, 7 branes,
- And these 3+1-d intersections
- Get gauge forces and gravity too

Either scenario

- Reason to have coincident 3-branes and 7-branes
- Even better—slightly mismatched generates hierarchy!
- Cosmology interesting: linking different horizon-sized regions
- Brane inflation possible—3-brane annihilations

Conclusions

- New way of thinking about cosmology
- Many new and interesting questions
- Two aspects of our discussion
- First fairly robust and remarkable
- We put in very little but 3-branes and 7-branes special
- Show up all over the place in string-theory
- 3+1-dimensions really is special
- Second part: new cosmologies with localized gravity
- Ripe for further exploration
- Dynamical relaxation principle very satisfying-merits further study