A Low Quadrupole from Inhomogeneous Dark Energy

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Possible ways to falsify:

- Show $w \neq -1$.
- Show dark energy interacting, e.g. fifth force, evolution of varying fine structure constant.
- Show dark energy not homogenous.
Scalar Field Dark Energy (Quintessence)

- Satisfies the Klein-Gordon equation

\[ \ddot{Q} + 3H \dot{Q} + V' = 0. \]

- If

\[ \frac{1}{M_p^2} \left( \frac{V'}{H} \right)^2 \ll 1, \quad \left| \frac{V''}{H^2} \right| \ll 1 \]

then

\[ \dot{Q} \approx 0 \]

and

\[ \rho_Q \approx V \approx -p \]

giving

\[ w_Q \approx -1. \]
Quintessence Perturbations

\[ \delta \ddot{Q} + 3H \delta \dot{Q} + \left( \frac{k^2}{a^2} + m^2 \right) \delta Q = 0. \]

Then

\[ \delta Q \approx \begin{cases} \text{constant}, & (k/a)^2, m^2 < H^2 \\ 1/a, & (k/a)^2, m^2 > H^2 \end{cases} \]

Therefore, no appreciable perturbations on scales smaller than the Hubble length.
Sachs Wolfe Effect: \( \frac{\Delta T}{T} = \frac{1}{3} \Phi \).
- **Sachs Wolfe**
  Effect: \( \frac{\Delta T}{T} = \frac{1}{3} \phi \).  

- **Spherical Harmonic** \( C_\ell \):  
  \( \theta \approx \frac{200}{1+\ell} \) deg.  

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**Projection**

- Quadrupole Angle
- Distance / \( \frac{1}{H_0} \)
- Sachs Wolfe Effect: \[ \frac{\Delta T}{T} = \frac{1}{3} \Phi. \]

- Spherical Harmonic \( C_\ell \): \[ \theta \approx \frac{200}{1+\ell} \text{ deg.} \]

- Integrated SW Effect: \[ \frac{\Delta T}{T} = 2 \int_0^1 \frac{\partial \Phi}{\partial z} \bigg|_{\text{geodesic}} dz \]
$P(C_2 \leq C_{2}^{\text{obs}} | \Lambda CDM) = 0.7\%$ (Spergel et al. (2003)). However, improved treatment of low $\ell$ likelihood (Efstathiou 2003; Slosar and Seljak 2004):

$P(C_2 \leq C_{2}^{\text{obs}} | \Lambda CDM) \approx 3\%$. 
$S_i = 0, -5, -10, -15, -20$
Statistical Significance

Assuming scale invariant primordial power spectrum (Gordon, Hu 2004) $P[\text{correlated} | \text{WMAP}] = 3.5\%$. However using improved likelihood for low $\ell$:
$P[\text{correlated} | \text{WMAP}] = 7.2\%$. But (de Oliveira-Costa, Tegmark, Zaldarriaga and Hamilton 2003):

\[ \langle (T + \text{foreground})^2 \rangle = \langle T^2 \rangle + \langle (\text{foreground})^2 \rangle. \]
Polarization
(Gordon and Hu 2004)

- Anticorrelated quintessence perturbations imply temperature/polarization correlation is suppressed at quadrupole scales while polarization is not.

- Current error bars on polarization/temperature correlation about twice cosmic variance. Polarization power spectrum no yet measured at quadrupole scales.
Inflationary Context

- Moroi and Takahashi (2003): 3 field curvaton model

- Variable decay, Dvali, et. al. (2004) and Kofman (2003):
  \[ \Phi_i = -\frac{1}{10} \frac{\delta \Gamma}{\Gamma}. \]

- For coupling \( \mathcal{L} = \cdots + \frac{Q}{M_p} \phi q \bar{q} + \cdots \) we have \( \Gamma \sim \left( \frac{Q}{M_p} \right)^2 m_\phi \)
  which gives \( \Phi_i = -\frac{1}{5} \frac{\delta Q}{Q}. \)
\[
\frac{\delta \rho_Q}{\rho_Q} \approx 6 \times 10^{-4}
\]
\[
= \frac{V'_Q}{V_Q} \delta Q
\]

which gives

\[
\delta Q \approx \frac{4 \times 10^{-4} M_p}{\sqrt{\epsilon_Q}} = \frac{H_{\text{inf}}}{2\pi}
\]

Therefore, Gravity waves 30 times too large.
Tachyonic Amplification
(Gordon and Wands 2005)

\[ \delta Q_f = \frac{r_f}{r_i} \delta Q_i . \]

After reheating, quintessence is a Pseudo-Nambu-Goldstone Boson (Frieman, et al. 1995).
Conclusions

- Perturbations in quintessence only on scales larger than Hubble length.
- Thus, would only effect low $\ell$ harmonics.
- Quadrupole observed to be low.
- Lower prediction for quadrupole if quintessence has perturbations anti-correlated with matter perturbations.
- Fit not statistically significant but could be improved with more polarization data.
- To get large enough quintessence perturbations from inflation a tachyonic amplification mechanism is needed.